Modeling of thermomechanical behaviour of embankment dams; one-component vs. multicomponent description

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Abstract

The main aim of this note is to present in juxtaposition continuous one-component and two-component models of geomaterials appearing in construction of embankment dams. In particular such features as saturation with water and seepage problems, modeling of fluidization yielding piping, generalizations of the Darcy law and changes of porosity are presented.

Introduction

Inspection of textbooks and manuals for civil and geotechnical engineers reveals that the design of embankment dams and levees is still based on two issues. It is either a stability analysis based on the one-dimensional Mohr-Coulomb relation

$$\tau = c + \sigma \tan \phi, \tag{1}$$

where τ is the shear strength, σ denotes the normal effective stress on the failure plane and c, ϕ denote the cohesion intercept and the friction angle, respectively, or these are flow nets and streamlines obtained by a graphical, for instance Schmidt's, method (e.g. see Figure 1).

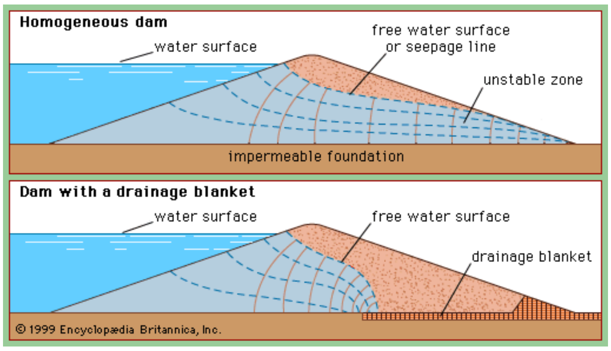

Figure 1. An example of flow nets for two types of embankment dams

Sometimes it is supplemented by Darcy's law for the estimation of seepage. The rest of those books contains hundreds of examples of existing constructions, a description of their behavior under various loading conditions and failures. Based on this empirical knowledge some heuristic hints for designers are formulated.

This is very different from engineering books in other branches of civil engineering where the design is based on theoretical models which have replaced a sheer collection of observations. This yields as well a very fruitful development of software for computer aided design, such as CAD, Novapoint, etc. M. and I. VANICEK [22] wrote in their book: 'When we look back on the whole process through which the geotechnical engineer has to go, we arrive at the conclusion that the degree of accuracy is significantly lower than for steel or concrete structure, where the differences in the design can be in the order of a few percent, while for earth structures these differences can be in order of tens of percent. That is why on one side the excellent knowledge of soil behavior and treatment of soil as construction material can bring significant savings against conventional design but on the other one disregarding this can lead to failures of earth structures.'

However, the situation is slowly changing to the better because the research in the field of soil mechanics has made a tremendous progress and many theoretical issues such as failure criteria, fluid flow in porous and granular media, heat transfer in soils, micro-macro transitions in theoretical modeling which incorporate porosity changes, saturation, phase changes, dynamics and, particularly, thixotropy or sound wave propagation in soils and rocks were successfully developed.

A choice of theoretical descriptions of aquifers, embankments and many other geotechnical structures depends on the class of phenomena which we want to embrace and on conditions in which the construction or its part should work. For instance, a mechanical loading of a granular dry material yields fragmentation and abrasion. The same mechanical loading of a water saturated granular material yields diffusion, fragmentation but much less abrasion. Hence, in the first case we may expect considerable changes of porosity while in the second case changes of permeability, piping, particle segregation etc. play an important role. This means that the water content in a geomaterial may essentially influence the choice of the theoretical description which is needed.

In this work we present a juxtaposition of the two fundamental approaches to the theoretical description of thermomechanical behavior of geotechnical materials. On the one hand-side, we sketch a one-component model with additional internal variables. This may be appropriate for the description of plastic behavior of geomaterials, abrasion but, in many cases of practical interest, also for the description of diffusion. On the other hand, we present a two-component model of a saturated granular material. This model contains a number of additional variables which are able to describe such phenomena as diffusion with variable permeability, localization of deformation (e.g. on filters) and internal erosion processes. This yields a theoretical description of the backward erosion, concentrated leak and suffusion which are, in turn, main reasons for piping.

One-component modeling of geomaterials

The origin of the one-component models of geomaterials stems from the classical model of elastoplastic materials. They belong to two groups: one describing dry granular materials driven by elastic properties and frictional interactions of grains and the other one describing fully saturated granular materials in which viscosity rather then friction contributes to the mechanical response of the system. Both classes of models contain the macroscopic deformation $\mathbf{B} = B_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$ (the left Cauchy-Green deformation tensor), the velocity $\mathbf{v} = v_i\mathbf{e}_i$ and the temperature θ as unknown functions of the position $\mathbf{x}$ and time t. However, they differ in the set of unknown microstructural variables. The first class contains only the roughness a whose time derivative $\dot{a}$ is called the abrasion, while the second class may contain the abrasion but it must contain also the porosity n and the pore pressure p. For these quantities – fields, we have to construct additional equations.
The classical approach is based on the set of conservation laws of mass, momentum and energy. In Cartesian reference frame they have the following form

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0, \tag{2}$$

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_k v_i - \sigma_{ik}) = \rho b_i, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_k v_k + \rho\varepsilon\right) + \frac{\partial}{\partial x_k}\left(\left(\frac{1}{2}\rho v_i v_i + \rho\varepsilon\right) v_k + q_k - \sigma_{ki} v_i\right) = \rho b_k v_k, \tag{4}$$

where ρ is the bulk mass density, σ_{ik} are components of the Cauchy stress tensor, $\mathbf{T} = \sigma_{ik}\mathbf{e}_i \otimes \mathbf{e}_k$, b_i are the body forces (e.g. gravitational or centrifugal), $\mathbf{b} = b_k\mathbf{e}_k$, ε is the specific internal energy and q_k are components of the heat flux vector, $\mathbf{q} = q_k\mathbf{e}_k$. It is often assumed that the real grains of the material are incompressible. This means that the true mass density ρ^{SR}, $\rho = (1-n)\rho^{SR}$ (S for 'solid' and R for 'real' or 'true'; in soil mechanics one denotes sometimes $\rho^{SR} = \gamma$), is constant. The porosity n is related to the void ratio e, $0 \leq e < \infty$, often used in soil mechanics, by the simple relation $n = e/(1+e)$, $0 \leq n \leq 1$. Then the mass conservation yields the relation for changes of n

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x_k}(n v_k) = 0. \tag{5}$$

In order to obtain the equations of the model of dry granular materials we have to specify constitutive relations for stress tensor σ_{ik}, internal energy ε, heat flux q_k and the abrasion $\dot{a}$. Experience shows that granular materials behave plastically. Consequently, the classical Mohr-Coulomb relation has been extended to relate the stress tensor and the deformation tensor. As the so-called hardening effects play an important role in such models one had to introduce additional internal variables (the so-called back-stress). The result is the cam-clay model commonly used in the literature on soil mechanics (e.g. see: D. MUIR WOOD [28], [29], LANCELLOTTA [18]). As an alternative a so-called hypoplasticity was introduced (BAUER [3], WOLFFERSDORF [27], KOLYMBAS [16], [17]). In contrast to the cam-clay model the hypoplasticity is rate-dependent which means that the rate of deformation $D_{ij} = \frac{1}{2}[\partial v_i/\partial x_j + \partial v_j/\partial x_i]$ (it is related to the time derivative of the deformation tensor B_{ij}) has an influence on the current values of the stress. The general constitutive relation for stresses in this model has the form

$$\mathring{\sigma}_{ij} = f_{ij}[\sigma_{ij}, D_{ij}, n], \tag{6}$$

where $\mathring{\sigma}_{ij}$ denotes an objective time derivative of the stress tensor.
We do not need to go into details of these models in the present work. Many of them can be found in the contribution of BAUER [4] to the first conference of this series (i.e. [35]).
It remains to specify the internal energy ε, the heat flux q_k and the abrasion $\dot{a}$. For the first two quantities one usually assumes that the classical Fourier model of heat conduction is valid. This may be questionable in some fast processes but for the thermomechanical description of embakments it seems to be sufficient. The distribution of temperature follows in the theoretical model from the energy conservation law (4). For the purpose of our analysis it is sufficient to point out the ways in which energy is transported in the medium. They are specified by contributions under the div operator (i.e. $\frac{\partial}{\partial x_k}$). The first one is the convection. The second one, described by $\mathbf{q}$, consists of two parts: the conduction, $\mathbf{q}_c$ and the diffusion, $\mathbf{q}_d$ with $\mathbf{q} = \mathbf{q}_c + \mathbf{q}_d$. The latter means that the energy is transported by the relative motion of components. We do not go into any details of this mechanism in this work. The conduction is related to the transfer of energy due to the temperature gradient. Finally, the last contribution, $\mathbf{Tv}$, is the bulk working of stresses which is also of no interest in this work.
Conduction in isotropic materials is usually described by the linear Fourier law

$$\mathbf{q}_c = -\lambda \mathrm{grad}\theta, \tag{7}$$

where θ is the absolute temperature. The coefficient λ, the heat conductivity, is for soils heavily dependent on the morphology. In Figure 2 (compare: FAROUKI [10]) we show an example of a nomogram in which the dependence of λ is demonstrated for various moisture contents (i.e. for various mass densities of the liquid if the mass density $\gamma_d = \rho^S$ of the skeleton is fixed), saturations (i.e. the volume fraction of the gas to the liquid component) and mass densities of the skeleton. As we see, in this example λ varies between 0.1 to 1.4 W/mK.

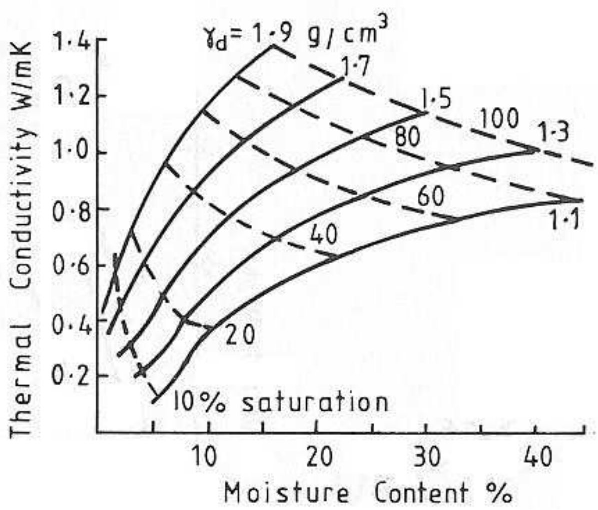

Figure 2. An example of nomograms for the heat conductivity in dependence on the soil density, saturation and moisture content

Unfortunately, the heat conductivity, λ, cannot be derived by means of any averaging procedure from microscopic conductivities of components. For this reason, we have to rely on empirical relations. These were proposed for soils since some 30 years.

Recently developed experimental equipment such as TP O2 probe allow to make non-steady-state measurements of heat conductivity (e.g. A. GONTASZEWSKA [24]) for various morphologies of soils. In the work, published in 2008, SHAN XIONG CHEN [7] has proposed the following empirical relation for the conductivity

$$\lambda = \lambda_0^{1-n} \lambda_w^n \left[(1-b) S + b \right]^{cn}, \tag{8}$$

where S is the saturation, λ_0 is the grain heat conductivity, $\lambda_w = 0.61$ [W/mK] – heat conductivity of water and b, c are fitting parameters. For example, for sandy soils $\lambda_0 = 7.5$ [W/mK], $b = 0.0022$, $c = 0.78$. In an implicit way, this relation accounts as well for a dependence on temperature through λ_0 and λ_w.

The results for the heat conductivity and corresponding theoretical one-component and multicomponent models play a particularly important role in description of freezing and frost heaving of soils.

The form of the equation for abrasion has a long history and it goes back to the work of GOODMAN and COWIN [13]. However, in this pioneering work the equation was proposed rather for changes of volume fraction than for the abrasion. It was first the series of works of K. HUTTER (e.g. [23], [25]) and the PhD Thesis of N. KIRCHNER [15] where this equation was thermodynamically justified. Its form follows from the assumption that microstructural changes of the configuration caused by the abrasion must be accompanied by the so-called configurational forces. Then the abrasion $\dot{a}$, corresponding to the classical notion of momentum, must satisfy a balance law which is assumed to have the form

$$\rho k \frac{\partial^2 a}{\partial t^2} = \frac{\partial h_i}{\partial x_i} + \rho \left(l + f \right), \tag{9}$$

where k is the material parameter describing the resistance of the material to changes of its internal surface. According to KIRCHNER [15]: 'change of surface properties includes the smoothening of initially rough grain surface (that is, $\dot{a} < 0$) as well as the roughening of initially smooth grain surfaces (that is, $\dot{a} > 0$)'. h_i is the surface stress of abrasion. l and f are supply and production, respectively and the latter must be given by a constitutive law of its own.

The above sketched one-component model of dry geomaterials is often extended by an equation describing the flow of water through the saturated granular material. All equations described above are assumed to remain unchanged. The seepage process through the saturated material is supposed to satisfy some additional balance law which is justified experimentally. Such a justification goes back on works of DARCY [9] and it has been incorporated in soil mechanics by VON TERZAGHI [21]. In the local form this law can be written in the form

$$Q_i = -\frac{k_{ij}}{\mu} \frac{\partial p}{\partial x_j}, \tag{10}$$

where Q_i is the so-called specific discharge, i.e. a relative velocity of water with respect to the skeleton, k_{ij} is the matrix of permeability which reduces to a scalar k for isotropic materials and μ is the kinematic viscosity of water. It has been shown that such a relation holds for small relative velocities which are connected with a laminar flow of the water (low Reynolds number, $Re < 1 \div 10$, J. BEAR [5]). In the case of fast flows yielding turbulence (high Reynolds numbers) the Darcy law does not hold. Most likely FORCHHEIMER [11] was the first who proposed nonlinear corrections to (10) in order to describe such flows. They play a particularly important role in processes of piping, commonly appearing in embankment dams. A more rational procedure of description of seepage is proposed by theories of multicomponent systems.

Two-component modeling of geomaterials

The thermomechanical model of a one-component geomaterial can be considerably improved when one applies a theory of immiscible mixtures. We shall do so for fully saturated materials. In this case the extension yields a better physical insight but for purposes of geotechnics it is not necessary. However, for partially saturated materials such extensions are unavoidable and, simultaneously, they are similar to two-component models in many technical details. As the space of this article does not allow for the extensive treatment of immiscible mixtures of many components we limit our attention only to two components: solid and water. Some properties of a three-component model of an unsaturated material are discussed in the work of ALBERS and WILMANSKI [2] in this Volume.

In the case of two components one has to describe the partial macroscopic fields for each component. These are partial mass densities ρ^S, ρ^F with the bulk mass density $\rho = \rho^S + \rho^F$, partial velocities $\mathbf{v}^S$, $\mathbf{v}^F$ with the bulk (barycentric) velocity $\mathbf{v} = \left(\rho^S/\rho\right)\mathbf{v}^S + \left(\rho^F/\rho\right)\mathbf{v}^F$, partial Cauchy stresses $\mathbf{T}^S = \sigma^S_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$, $\mathbf{T}^F = \sigma^F_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$ with the bulk stress $\mathbf{T} \approx \mathbf{T}^S + \mathbf{T}^F$. These quantities must satisfy balance laws

$$\frac{\partial \rho^\alpha}{\partial t} + \frac{\partial}{\partial x}\left(\rho^\alpha v_i^\alpha\right) = 0, \quad \alpha = S, F, \tag{11}$$

$$\frac{\partial\left(\rho^\alpha v_i^\alpha\right)}{\partial t} + \frac{\partial}{\partial x_k}\left(\rho^\alpha v_i^\alpha \otimes v_k^\alpha\right) = \tag{12}$$

$$= \frac{\partial \sigma^\alpha_{ik}}{\partial x_k} + \hat{p}_i^\alpha + \rho^\alpha b_i^\alpha, \tag{13}$$

$$\hat{p}_i^S + \hat{p}_i^F = 0.$$

It is easy to check that conservation laws (3), (4) of a one component model are then identically satisfied provided we neglect quadratic terms in relative velocities $\mathbf{v}^\alpha - \mathbf{v}$ which seems to be well justified for processes in soils far from the structural loss of stability such as fluidization. The momentum source $\hat{\mathbf{p}}^S = \hat{p}_i^S\mathbf{e}_i = -\hat{\mathbf{p}}^F = -\hat{p}_i^F\mathbf{e}_i$ is related to the diffusion force. In the case of an isotropic model linear in relative velocities it can be written in the form

$$\hat{\mathbf{p}}^S = \pi\left(\mathbf{v}^F - \mathbf{v}^S\right), \tag{14}$$

where π is the permeability coefficient. In the case of water one can assume that the partial stress tensor $\mathbf{T}^F$ is spherical, i.e. it reduces to the partial pressure p^F. Then the momentum balance for the fluid written in Cartesian coordinates has the form

$$\rho^F\left(\frac{\partial v_i^F}{\partial t} + v_k^F\frac{\partial v_i^F}{\partial x_k}\right) = -\frac{\partial p^F}{\partial x_i} + \pi\left(v_i^F - v_i^S\right) + \rho^F b_i^F. \tag{15}$$

This equation yields Darcy's law for processes with small changes of porosity and negligible inertial forces. In such a case, it follows

$$p^F = n_0 p, \quad \left(v_i^F - v_i^S\right) = -\frac{n_0}{\pi}\frac{\partial p}{\partial x_i}, \tag{16}$$

where n_0 is the initial porosity and $n \approx n_0$. This is identical with (10) for isotropic materials with an appropriate definition of the permeability $k/\mu = n_0/\pi$. Hence the two-component model yields the one-component model as a particular case.

On the other hand, the general form of partial momentum balance (13) admits also nonlinear contributions which yield the loss of stability of the fluid motion. If changes of porosity and relative velocity are not small one can introduce the following relation for the source of momentum (see: WILHELM, WILMANSKI [26])

$$\hat{p}_i^S = \pi\left(v_i^F - v_i^S\right) - \left(p + \rho^S\frac{\partial \psi^S}{\partial n}\right)\frac{\partial n}{\partial x_i}, \tag{17}$$

where p is the pore pressure and ψ^S is the Helmholtz free energy of the solid component dependent on deformations, porosity and relative velocity. This form of the source is justified by thermodynamic considerations which we shall not discuss in this work. The simplest choice of the dependence on the relative velocity which yields piping is as follows

$$\rho^S\frac{\partial \psi^S}{\partial n} = \frac{\Gamma}{\sqrt{2}}\left(1 + \frac{W - Y}{|W - Y|}\right)\sqrt{W}, \tag{18}$$

$$\Gamma, Y > 0, \quad W = \frac{1}{2}\left(v_i^F - v_i^S\right)\left(v_i^F - v_i^S\right),$$

where Γ is a material parameter and Y is the threshold velocity. As shown in [26] this model yields a quantitative agreement with experiments on sands.

Changes of porosity may be described by the relation (5) following from the assumption on incompressibility of grains. However, there is an evidence stemming from poroacoustics that such an assumption eliminates an important P2-wave from the model (e.g. WILMANSKI [33]). Consequently, one has to introduce an equation for porosity. As already mentioned the first attempt has been made by GOODMAN and COWIN [13] which was subsequently extended by Passman, Nunziato and Walsh [20]. This second order equation is still used but it seems to be more appropriate for abrasion than porosity. Another possibility is offered by a simple balance equation (WILMANSKI [30], [31])

$$\left(\frac{\partial}{\partial t} + v_i^S\frac{\partial}{\partial x_i}\right)\Delta_n + \frac{\partial}{\partial x_i}\left(\Phi\left(v_i^F - v_i^S\right)\right) = \hat{n}, \tag{19}$$

$$\Delta_n = n - n_{E,}$$

where Φ is a material parameter, n_E is the value of porosity in thermodynamical equilibrium and $\hat{n}$ is the source of porosity related to relaxation processes. This equation is thermodynamically admissible and yields a consistent model for large deformations (WILMANSKI [34]). In the case of small deformations of soils this equation can be immediately solved. Without relaxation processes ($\hat{n} = 0$) it yields the following relation

$$n = n_0\left(1 + \delta e + \frac{\Phi}{n_0}\left(e - \varepsilon\right)\right), \tag{20}$$

where n_0 is the initial porosity and both material parameters δ and Φ are given in terms of compressibilities of components. e and ε are macroscopic volume changes of the solid and fluid component, respectively. The same relation follows within the famous Biot model (e.g. BIOT [6]). The thermodynamic nonequilibrium contribution $\frac{\Phi}{n_0}\left(e - \varepsilon\right)$ is usually small. For sand saturated with water it is less then 10% of the first term. As a mater of fact the first paper on changes of porosity referring to microstructural properties of granular materials was written by F. GASSMANN in 1953 [12] (see: WILMANSKI [32] for the detailed discussion), and, in this paper, Gassmann proposed the first (equilibrium) part of the relation (20). On the other hand, the values of material parameter δ are such that only in the range of porosities smaller than app. 0.2 differences between the relation (20) and values following from the incompressibility assumption are visible. In Figure 3 we compare the values of δ following under the assumption of incompressibility (solid line) with those for two values of compressibility modulus 35 GPa and 48 GPa with air in pores – these two curves coincide with the line for incompressible case, and with water in pores – these are two lower curves (broken lines).

One can conclude the above remarks that the two-component model may play an important role in geotechnics for processes in which nonlinearities are essential. This concerns large deformations and, consequently, large changes of porosity and permeability. Of particular importance are, however, large relative (seepage) velocities which yield the loss of stability and piping. Otherwise one-component models seem to be acceptable for both dry and wet granular materials of geotechnical bearing.

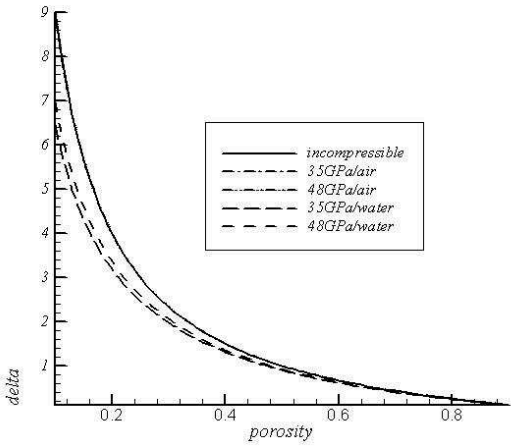

Figure 3. Values of material parameter δ for dry and wet material with two values of compressibility modulus of grains and for incompressible grains

Concluding remark

Three important issues of theoretical modeling should be mentioned.

The first one is the formulation of boundary conditions. In one-component models these are classical and extensivelly discussed in elasticity or plasticity. In multicomponent models the problem is more complicated because one has two formulate additionl conditions for the extended set of partial differential equations. Even in the case of impermeable boundaries and such are phreatic surfaces of contact between saturated and dry domains of soils one has to formulate an equation of motion of the surface itself. Such moving boundaries yield the boundary value problems with free boundaries and these are usually ill-posed and create big mathematical problems. A physical presentation of this problem can be found in the book of BEAR [5]. The situation is even worse on permeable surfaces. A part of the conditions on such surfaces has been formulated by von Terzaghi who had shown that the external loading must be taken over by the whole stress vector $\mathbf{Tn} = \sigma_{ik}n_k$ where $\mathbf{n} = n_k\mathbf{e}_k$ is the unit normal vector of the boundary. The second condition was extensively discussed in the literature and it concerns the flow through the boundary. This boundary condition for inviscid fluids relates the pressure difference and the velocity of flow through the surface. It contains an additional material parameter, the so-called surface permeability. It plays a very important role on contact surfaces between different layers saturated with water and on the external surface which is the seepage face.

The second issue appears if the transition zones of not fully saturated soils appear. They are created, for example, by infiltration processes. In such processes one has to account for the capillary effects and an appropriate theoretical model must describe more than one fluid component. It is only in recent few years that such models are developed. Presentation of a linear three-component model with cappilary effects in applications to poroacoustics can be found in the book of B. ALBERS [1].

The third important issue is the development of software for geotechnical engineers which would account for all those theoretical problems which we have mentioned above. Such computational packages do not exist yet. It is only very recent that the research in this direction has been intensified. In particular, it concerns the formulation of some macroscopic constitutive laws in terms of microscopic material properties in which the micro-macro transition would be done in a numerical way. H. MATTSSON, J. G. I. HELLSTRÖM, S. LUNDSTRÖM [19] formulate this problem in their extensive survey work from the year 2008 in the following manner: 'The main objective with this literature survey is to elucidate the state of the art of internal erosion in embankment dams in order to be able to formulate a research program for numerical modelling of internal erosion in a physically sound manner.'

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