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Thermodynamic Modelling of Soil Morphology

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Abstract

Macroscopic modeling of soils is based on a number of properties which refer to the mesoscopic morphology. The most fundamental parameters of this art are

1. coupling parameters between partial stresses of components and deformations of components,
2. porosities
3. saturation
4. permeability and diffusivity, tortuosity.

They are discussed in this note.

Introduction

Thermodynamic modeling of soils develops as a brunch of the theory of immiscible mixtures. This modern continuous approach to systems with multicomponent structures whose one component is solid has been initiated by works of R. M. Bowen (e.g. [1]). Numerous nonlinear effects in porous and granular materials yield in such a modeling a necessity of application of rather complex mathematical tools. This complexity is then hidden in modern computer programs whose applications in engineering do not require high mathematical skills from software users.

However, such thermodynamic models of soils require as well an identification of various quantities which do not usually appear in classical continuous models. This concerns, in particular, geometrical properties of such systems, true properties of their components and some process variables characteristic for systems with microstructure. We call this identification the modeling of soil morphology. To name a few examples, one has to identify the porosity, saturation, moisture, capillary pressure, permeability and many others. This leads very often to confusion and misinterpretation of results. For this reason, in this work we show fundamental quantities of thermodynamic models of immiscible mixtures appearing in description of soils and their relation to quantities commonly used by soil engineers.

Micro-macro transitions for porous materials

The construction of thermodynamic models of materials with small randomly distributed voids requires always a smearing-out procedure which transforms functions on complicated and different domains to a common domain, say $\mathcal{B}_0$, which, in turn,

is identified with a continuous porous medium. Such constructions for multicomponent systems with a relative motion (diffusion) yield the so-called immiscible mixtures. Quantities appearing in such models are not directly measurable and require certain identification rules. In some cases these rules can be simply constructued by volume averaging and in some other cases they require more sophisticated methods of identification. We begin our presentation with a few typical quantities which follow from the former procedure.

The volume averaging is performed by means of the so-called Representative Elementary Volume (REV, e.g. [2]) consisting of points $\mathbf{Z}$. REV is small enough to be replaced by a material point $\mathbf{X}$ of $\mathcal{B}_0$ at which a set of macroscopic quantities replaces real (true) quantities of the microdomain REV. For three-component systems which we consider in this work, the following quantities are defined by averaging over $REV\,(\mathbf{X}, t)$ prescribed to the point $\mathbf{X} \in \mathcal{B}_0$ at the instant of time t:

1. Porosity

$$n\,(\mathbf{X}, t) = 1 - \frac{1}{\mathrm{vol}\,(REV)} \int_{REV(\mathbf{X},t)} \chi^S\,(\mathbf{Z}, t)\, dV_Z, \quad (1)$$

where $\chi^S\,(., t)$ is the characteristic function of the skeleton, i.e. it is equal to one if the point $\mathbf{Z}$ is occupied by a particle of the skeleton (solid phase) at the instant of time t, and zero otherwise, and $\mathrm{vol}\,(REV) = V$ is the volume of REV. In continuous models, the porosity is usually identical with the porosity available for the transport in pores. This means that, for instance, contributions of dead-end channels are not included in n. The latter contribute to changes of the true (real) effective properties of the skeleton.

2. Mass density of the skeleton

$$\rho^S\,(\mathbf{X}, t) = \frac{1}{\mathrm{vol}\,(REV)} \int_{REV(\mathbf{X},t)} \rho^{SR}\,(\mathbf{Z}, t)\, \chi^S\,(\mathbf{Z}, t)\, dV_Z, \quad (2)$$

where $\rho^{SR}\,(\mathbf{Z}, t)$ is the real (true) mass density of the skeleton at the point $\mathbf{Z}$ and at the instant of time t.

3. Mass density of the liquid

$$\rho^L\,(\mathbf{X}, t) = \frac{1}{\mathrm{vol}\,(REV)} \int_{REV(\mathbf{X},t)} \rho^{LR}\,(\mathbf{Z}, t)\, \chi^L\,(\mathbf{Z}, t)\, dV_Z, \quad (3)$$

where $\rho^{LR}(\mathbf{Z}, t)$ is the real (true) mass density of the liquid at the point $\mathbf{Z}$ and at the instant of time t, and $\chi^L(., t)$ is the characteristic function of the liquid, i.e. it is equal to one if the point $\mathbf{Z}$ is occupied by a particle of the liquid at the instant of time t, and zero otherwise.

4. Mass density of the gas

$$\rho^G(\mathbf{X}, t) = \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{X}, t)} \rho^{GR}(\mathbf{Z}, t) \chi^G(\mathbf{Z}, t) \, dV_Z, \tag{4}$$

where $\rho^{GR}(\mathbf{Z}, t)$ is the real (true) mass density of the gas at the point $\mathbf{Z}$ and at the instant of time t, and $\chi^G(., t)$ is the characteristic function of the gas, i.e. it is equal to one if the point $\mathbf{Z}$ is occupied by a particle of the gas at the instant of time t, and zero otherwise.

In many cases of practical importance, one assumes a micro-homogeneity of the microstructure which means that true mass densities are approximately constant on REV. Then

$$\rho^S = (1-n)\rho^{SR}, \quad \rho^L = nS\rho^{LR}, \tag{5}$$

$$\rho^G = n(1-S)\rho^{GR}, \quad n = \frac{V_L + V_G}{V}, \tag{6}$$

$$V_\alpha = \int_{REV(\mathbf{X}, t)} \chi^\alpha(\mathbf{Z}, t) \, dV_Z,$$

where S denotes the saturation

$$S = V_L / (V_L + V_G), \tag{7}$$

and $\rho^{SR}, \rho^{LR}, \rho^{GR}$ are evaluated in an arbitrary point $\mathbf{Z}_0 \in REV(\mathbf{X}, t)$ (microhomogeneity!).
Sometimes the product

$$\theta = nS \equiv V_L/V, \quad V = V_S + V_L + V_G, \tag{8}$$

is called the volumetric water content. This should be distinguished from the gravimetric water content (a moisture fraction)

$$w \equiv m_W = M_L / M_S = \rho^{LR} V_L / (\rho^{SR} V_S), \tag{9}$$

$$V_L = nSV, \quad V_S = (1-n)V,$$

i.e. it is the fraction of the mass of liquid to the mass of the (real, true, dry) skeleton. Certainly, bearing relations (5) in mind, it can be written in the form

$$m_W = \rho^L / \rho^S. \tag{10}$$

Consequently, in contrast to porosity, n, and saturation, S, neither the volumetric water content, θ, nor the moisture fraction, m_w, are independent microstructural quantities. However, the latter is sometimes used as a measure of compactness of soils reflected by the partial mass density of the skeleton, ρ^S.
The construction of volume averages illustrated above fails in the case of transport coefficients. This concerns both classical coefficients such as heat conductivity or hydraulic conductivity, as well as partial stresses. In the latter case, one makes sometimes simplifying assumptions

$$p^S = (1-n)p^{SR}, \quad p^L = nSp^{LR},$$

$$p^G = n(1-S)p^{GR}, \quad p^F = p^L + p^G = np^{FR}, \tag{11}$$

$$p^{FR} = Sp^{LR} + (1-S)p^{GR},$$

where $p^S = -1/3\text{tr}\mathbf{T}^S$ is the partial pressure in the skeleton ($\mathbf{T}^S$ denotes the partial Cauchy stress tensor in the skeleton) and p^{SR} denotes the true (real) pressure in the skeleton, p^L and p^{LR} are partial and, respectively, real pressure in the liquid, p^G and p^{GR} – in the gas, and p^{FR} is the pore pressure. One can show that relations (11) are a combination of volume averaging under very restrictive conditions and Dalton's law for fluid components. However, one should bear in mind that they may not hold, for instance, near boundaries or for dynamical processes such as shock waves. The following quantity

$$p_c = p^{GR} - p^{LR}, \tag{12}$$

is identified with the capillary pressure and used in constitutive relations for the saturation, S.

Fields of a continuum model

As indicated in the previous Section, we limit the attention to the class of three-component continuous models. Then, the aim of thermodynamical analysis is the construction of equations for the following fields

1. partial mass densities, ρ^S, ρ^L, ρ^G,
2. velocities of components, $\mathbf{v}^S, \mathbf{v}^L, \mathbf{v}^G$,
3. common temperature of components, T,
4. porosity, n,
5. saturation, S.

The physical significance of the first group has been explained above.
Fields of velocities of components are more difficult to interpret in terms of microstructural quantities. It is obvious that deviations of real velocities of the skeleton and of the fluid components from their macroscopic counterparts, both with respect to directions and the magnitude, are usually very large. Such fluctuations cannot be easily estimated by averaging. Hence, the terminology: diffusion velocities $\mathbf{v}^L - \mathbf{v}^S$, $\mathbf{v}^G - \mathbf{v}^S$, with respect to the skeleton – or frequently appearing in soil mechanics – seepage velocity or velocity of filtration should be understood as macroscopic notions and their interpretation may change from one process to the other. For instance, in cases of catastrophic phenomena such as liquefaction the filter velocity cannot be identified with any of the macroscopic average velocities. Macroscopic models based on the above notions must be then correspondingly extended.
At the first glance, the problem of averaging of velocities can be replaced by the averaging of momenta. These are volume densities and, consequently, one could write

$$\rho^S \mathbf{v}^S = \frac{1}{\text{vol}\,(REV)} \int\limits_{REV(\mathbf{X},t)} \rho^{SR}\,(\mathbf{Z},t)\,\mathbf{v}^{SR}\,(\mathbf{Z},t)\,\chi^S\,(\mathbf{Z},t)\,dV_Z,$$

$$\rho^L \mathbf{v}^L = \frac{1}{\text{vol}\,(REV)} \int\limits_{REV(\mathbf{X},t)} \rho^{LR}\,(\mathbf{Z},t)\,\mathbf{v}^{LR}\,(\mathbf{Z},t)\,\chi^L\,(\mathbf{Z},t)\,dV_Z,$$

$$\rho^G \mathbf{v}^G = \frac{1}{\text{vol}\,(REV)} \int\limits_{REV(\mathbf{X},t)} \rho^{GR}\,(\mathbf{Z},t)\,\mathbf{v}^{GR}\,(\mathbf{Z},t)\,\chi^G\,(\mathbf{Z},t)\,dV_Z,$$

$$(13)$$

where $\mathbf{v}^{SR}, \mathbf{v}^{LR}, \mathbf{v}^{GR}$ denote real (true) velocities in channels. However, this is not much of the help. First of all, the macrocsopic momenum balance equations which yield field equations for $\mathbf{v}^S, \mathbf{v}^L, \mathbf{v}^G$ must contain source terms such as diffusive forces reflecting interactions through walls of channels or frictional forces between fluid components. Such terms are not present in microstructural momentum balance equations and must be introduced by some additional surface or line integrals on REV which, of course, has nothing to do with volume averaging. Secondly, average momentum densities do not describe strong deviations (fluctuations) of true momenta from average values anyway. These would have to be introduced additionally to averaged momentum balance equations. Consequently, in spite of a few attempts to develop this procedure it seems to be easier to deal directly with purely macroscopic models.

One of the important quantities which we present further in this work is the tortuosity, τ. It does not appear among the fields because there exists no model with a corresponding field equation. We return to this problem later. Also the fields of porosity, n, and saturation, S, are discussed below in some details. Field equations for the above fields follow either from balance equations or from additional constitutive assumptions (saturation). Balance equations contain fluxes and sources and these, in turn, require additional information on material parameters. In our present work we address three such parameters: the sources of momenta, porosity and heat conduction.

Porosity

The field of porosity, n, is a typical microstructural (internal) variable. In many linear models used in soil mechanics (e.g. soil acoustics) it is assumed to be constant. However, many problems must account for changes of porosity. For instance, within soil mechanics these are large plastic deformations (e.g. [3]), damage (for instance, in freezing and thawing) or, for granular materials, combustion problems of solid fuels. Generally, there are a few different approaches to changes of porosity. We mention here the five most commonly appearing in the literature:

1. Constitutive assumption, e.g. equilibrium changes coupled on volume changes of the solid (Gassmann [4]),
2. Incompressibility assumption (Bowen [5]),

3. Evolution equation (Bowen [6]),
4. Second order equation based on a principle of equilibrated pressure (Goodman and Cowin [7], Hutter et al. [8]),
5. Balance equation (Wilmanski [9], [10]).

Gassmann's model follows from a simplified micro-macro description and results in the relation

$$n = n_0\,(1 + \delta e)\,, \tag{14}$$

where e denotes small volume changes of the skeleton and δ is a material parameter related to compressibilities of components. n_0 is the initial porosity. We show some properties of this model further in this Section.

The assumption of incompressibility which is essential for Bowen's approach [5] states $\rho^{SR} = \text{const.}$, which, according to relations (5) yields the following form of the macroscopic mass balance equation for the skeleton

$$\rho^{SR}\frac{\partial}{\partial t}\,(1 - n) + \rho^{SR}\text{div}\,\big[(1 - n)\,\mathbf{v}^S\big] = 0, \tag{15}$$

provided there is no mass exchange with other components. Easy integration of this equation yields for small deformations

$$n = n_0\,[1 + (1 - n_0)/n_0\,e]\,, \tag{16}$$

which, obviously, reminds Gassmann's relation (14) but there is no relation to material parameters. Again, we return to this relation further in this Section.

We will not discuss the next two models and mention only that the evolution equation proposed by Bowen is a particular case of the balance equation of porosity, at least for small deformations, when one neglects the influence of diffusion. The Goodman and Cowin proposition is related to some microstructural considerations which have a bearing in the case of combustion problems for powders. Hutter and his coworkers have shown that some extensions of this model describe well the behavior of avalanches.

The last model, mentioned above, has been developed primarily for applications under large deformations. Its linear version for two-component systems has the form

$$\frac{\partial \Delta_n}{\partial t} + \Phi\text{div}\,\big(\mathbf{v}^F - \mathbf{v}^S\big) = -\frac{\Delta_n}{\tau},$$
$$\Delta_n = n - n_E, \quad n_E = n_0\,(1 + \delta e)\,, \tag{17}$$

where $\mathbf{v}^F$ is the macroscopic velocity of the fluid, τ, δ, Φ are material constants. In the case of soils, one can usually neglect relaxation effects described by the right-hand side of the equation. This means that one can take the limit $\tau \to \infty$. Then, the equation can be integrated and the following relation follows

$$n = n_0\,(1 + \delta e + \gamma\,(e - \varepsilon))\,, \quad \gamma = \frac{\Phi}{n_0}, \tag{18}$$

where ε is the volume change of the fluid component. The quantity $\zeta = (e - \varepsilon)/n_0$ is called the increment of fluid content. The above relation can be easily extended to three components (see: [11]). We shall not present this equation in this work.
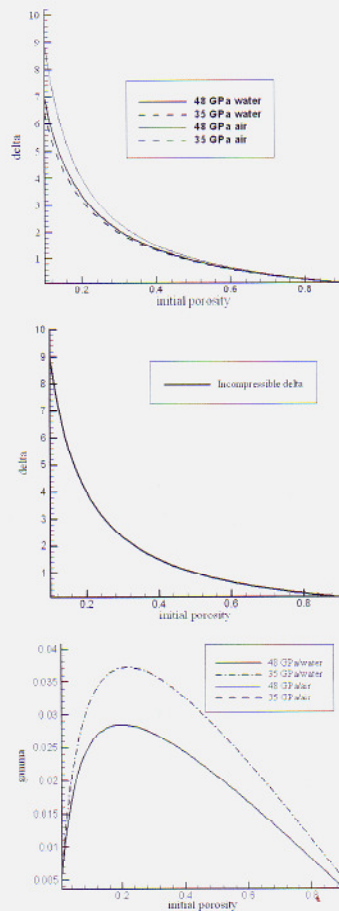

Figure 1: Coefficients δ and γ of the linear porosity relation without source. Top: coefficient δ for compressibilities of the skeleton 35 and 48 GPa with water or air filling the pores, middle: δ for incompressible skeleton ($\rho^{SR} = $ const.), bottom: coefficient γ for nonequilibrium contribution

Similarly to the Gassmann relation, it can be shown that material parameters δ, γ can be identified by means of compressibilities of components [12]. Simultaneously, they coincide with coefficients of porosity changes predicted by Biot's model [13]. In Figure 1, we present a few examples of the behavior of these parameters.

The upper panel shows the dependence of the coefficient δ (equilibrium changes of porosity) as a function of the initial porosity, n_0. The curves correspond to the compressibility of the solid skeleton 48 GPa and 35 GPa, respectively. The two lower curves are plotted for water and the two coinciding upper curves are plotted for air. The middle panel shows the illustration of δ for an incompressible skeleton (Equation (16); in this case $\gamma \equiv 0$). There is almost no difference between this curve and the curves for air in the upper panel. Finally, the bot-

tom panel shows the behavior of γ which reflects the influence of diffusion on changes of porosity. Obviously, this influence is rather small and the values for the saturation with air are so small that the curves are not visible in the scale of Figure 1.

Capillary pressure, saturation, retention curves

As mentioned earlier, the construction of a relation for the saturation as one of the fields for a three-component immiscible mixture is based on considerations concerning mechanical properties of the microstructure. They are still limited to isothermal conditions.

Between two immiscible fluids, of which one may be gaseous, e.g. water and air, a discontinuity in pressure exists across the interface separating them. The difference is called capillary pressure (compare Equation (12)) wherein p^{GR} is the true pressure in the nonwetting phase (gas in our case) and p^{LR} the corresponding value in the wetting fluid (water in our case). The fluid phase whose molecules or atoms preferentially are adsorbed on a solid surface is called the wetting fluid while the superseded material is denoted as nonwetting fluid.

In a fluid-gas-mixture, the Young-Laplace equation describes the capillary pressure difference due to the phenomenon of surface tension σ and relates it to the radius of the bubbles r ($p_c = 2\sigma/r$). For a porous medium, the capillary pressure is a measure of the tendency to suck in the wetting fluid or to repel the nonwetting phase. In soil science, the negative of the capillary pressure (expressed as the pressure head) is called suction. The radius r then is of the order of magnitude of the pore or grain size. The capillary pressure, thus, depends on the geometry of the void space, on the nature of the solids and fluids and on the degree of saturation. As we have already pointed out, in soils, the geometry of the void space is extremely irregular and complex. Hence, an idealized model may be adopted (e.g. capillary tubes, spheres of constant radius or a bundle of parallel circular rods). Laboratory experiments are probably the only method to derive the relationship $p_c = p_c(S)$.

In the experimentally determined capillary pressure curves a hysteresis occurs. This means that different capillary pressures may be obtained for a certain degree of saturation, depending on whether a sample is initially saturated with a wetting or with a nonwetting fluid. In both cases the fluid initially saturating the sample is slowly displaced by the other fluid. When the sample is initially saturated with a wetting fluid, the process is called drainage, otherwise imbibition. Figure 2 which is taken from [2] shows a typical capillary pressure - wetting fluid saturation relationship (kerosene and water in a sandstone) including the effect of hysteresis. In theoretical approaches the hysteresis is neglected for simplification. The capillary pressure curve is known also under several other denotations, e.g. it is also called retention curve, pF-curve or soil-water characteristic curve.
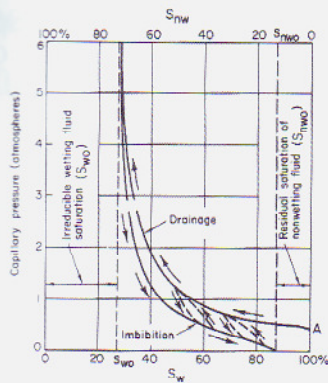
Figure 2: Typical capillary pressure - wetting fluid saturationd curves illustrating hysteresis [2]

Inspection of Figure 2 shows that a certain quantity of wetting fluid remains in the sample even at high capillary pressures. The value of the water saturation at this point (Bear denotes it by S_{wo} otherwise it is often described by S^{rF} – r stands for residual) is called irreducible saturation of the wetting fluid. Something similar appears if one looks at the imbibition curve. It is observed that at zero capillary pressure there remains a certain amount of the nonwetting fluid – the residual saturation of the nonwetting fluid (Bear denotes it by S_{nw0} otherwise it is often described by S^{rG}). It indicates the amount of entrapped air in the pores. This leads to the introduction of the term effective saturation (for more information see e.g. [14])

$$S_r := \frac{S - S^{rF}}{1 - S^{rF} - S^{rG}} \quad \text{or} \quad S_e := \frac{S - S^{rF}}{1 - S^{rF}}. \quad (19)$$

There are some attempts to put once experimentally measured curves into formulae. The most common approaches are those of Brooks & Corey [15] and van Genuchten [16]. According to the latter the relationship is described by

$$p_c = \frac{1}{\alpha_{vG}} \left[S_e^{(-1/m_{vG})} - 1 \right]^{1/n_{vG}}, \quad (20)$$

where $\alpha_{vG}, m_{vG}, n_{vG}$ are parameters which depend on the type of the soil and S_e is the effective water saturation (19). For simplicity mostly it is assumed that $S_e \equiv S$. Obviously, the van Genuchten equation is a nonlinear relation in the saturation S. It is clear that Equation (20) can be incorporated in a macroscopic model only under the condition that we know how to transfer true pressures p^{GR} and p^{LR} to the macroscopic level. The simplest possibility is to use relations (11) but they may be too simplistic as we have already indicated. They may give reasonable results in linear models but, in general, the problem is still open.

Tortuosity

The notion of tortuosity contributes to numerous confusions in modeling porous media. It is still disputable in what way one should include this measure of complexity of microstructure.

The tortuosity, τ, is taken as the ratio of the average pore length to the macroscopic characteristic length along, for instance, the major flow. An example of different value of tortuosity for a medium with the same local porosity is exemplified in Figure 3. The upper situation yields the smallest value of τ and the lowest situation – the largest value of τ among these three cases. The value of τ is the ratio of the distance of points A and B to the length of channels indicated by the arrows.
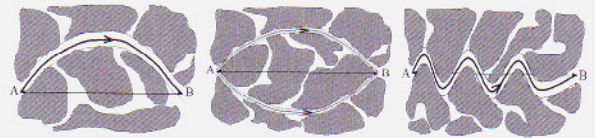

Figure 3: Different tortuosities for three morphologies of the same porosity

As shown in [17] the tortuosity influences the intensity of diffusion by entering a material parameter relating the pressure gradient and the diffusion velocity. We return to this coefficient in the next Section. However, it should be mentioned that it is the square of τ which appears in this relation. The error of the linear dependence made by Kozeny [18] has been corrected in many works and the quadratic dependence seems to be well established.

In many papers on acoustics of porous media it is claimed that the tortuosity enters the model by the so-called added mass effect. This has been introduced by Biot [19] in the form of a off-diagonal contribution ρ_{12} to the partial mass matrix. It can be understood as a coupling of components through inertial forces. One can show in a simple thermodynamic analysis [20] that such a coupling is nondissipative. This means that the tortuosity cannot have an influence on the damping of acoustic waves. This is, of course, a nonsensical conclusion. This contribution, can be indeed introduced to poroelastic models after some nonlinear corrections, but it cannot be interpreted as an influence of tortuosity.

Permeability

The notion of permeability of porous media is usually related with the Darcy law which expresses a total discharge of fluid in a one-dimensional flow in terms of the pressure difference. However, there is still a bit of confusion in the terminology. We use in the model of porous materials the term of a coefficient of permeability or coefficients of permeability if we deal with more than one fluid component. We proceed to introduce these notions in a systematic way.

The fundamental notion of intrinsic permeability, κ, is related solely to the morphology of a porous skeleton and it is independent of the kind of fluid in the channels. It may be related to the effective diameter of the pores, d, by the relation

$$\kappa = Cd^2, \quad (21)$$

where C is a dimensionless constant. The units of κ are $[\text{m}^2]$ but in practical applications often the unit 1 [darcy] $\equiv$ 1 D

$\approx 10^{-12}\,[\mathrm{m}^2]$ is used. In the second column of Table 1 we quote some typical values of this parameter.

TABLE 1: INTRINSIC PERMEABILITY, κ AND HYDRAULIC CONDUCTIVITY, K FOR SOME FORMATIONS

soil	κ [darcy]	K [m/s]
well sorted gravel	$10^5 - 10^2$	$1 - 10^{-3}$
oil reservoir	$10 - 10^{-1}$	$10^{-4} - 10^{-6}$
sandstone	$10^{-2} - 10^{-3}$	$10^{-7} - 10^{-8}$
granite	$10^{-6} - 10^{-7}$	$10^{-11} - 10^{-12}$

In order to characterize the flow of a particular fluid one has to account also for properties of this fluid. If the dynamic viscosity is μ and its mass density ρ^{LR} then the parameter

$$K = \kappa \gamma^{LR}/\mu, \quad \gamma^{LR} = \rho^{LR} g, \qquad (22)$$

is called the hydraulic conductivity; g is the earth acceleration. A few typical values for water (pressure 10^5 Pa, temperature 20^0 C, $\mu = 1.002 \times 10^{-3}$ Pa·s) are quoted also in Table 1. Then the typical form of the Darcy law is as follows

$$Q\left(\rho^{LR}g\right) = KA\Delta p/L, \qquad (23)$$

where Q $[\mathrm{m}^3/\mathrm{s}]$ is the total discharge through the surface A, $\Delta p = p_a - p_b$ is the pressure difference and L is the distance between two faces a and b. In addition, for horizontal aquifers one uses the notion of transmissivity, T, which is the product of the hydraulic conductivity, K, and the thickness, d, of the aquifer.

If we refer the diffusive flow to the difference of the concentration rather than to the difference of the pressure – one speaks then about the first Fick law – then the diffusion flux and the gradient of concentration are connected by the diffusion coefficient, D. This notion is, of course, related to the hydraulic conductivity, K, by a simple change of variables.

Finally, let us remark that the constant C of Equation (21) is dependent on the porosity and on the tortuosity (a quadratic dependence as we argued before).

Theories of porous media are based on the model of immiscible mixtures. Then Darcy's law does not enter the model at all. It is replaced by partial momentum equations. In the case of a linear model of two components the simplest form of this equation is the following

$$\rho^F \frac{\partial \mathbf{v}^F}{\partial t} = -\mathrm{grad}\, p^F + \pi \left(\mathbf{v}^F - \mathbf{v}^S\right). \qquad (24)$$

If we neglect the acceleration it becomes a precursor of Darcy's law. The coefficient π which is inversely proportional to the hydraulic conductivity is called the coefficient of permeability. The microstructural justification of the above relations for parameters of permeability is difficult. Such laws are known from the kinetic theory of mixture of gases and, in the case of granular materials, they are mimiced by assuming that grains have a very big mass in comparison to the the gas in pores and, consequently, can be assumed to be immobile. One obtains the so-called gas-dust diffusivity [21] which is the counterpart of the hydraulic conductivity. Some results, also experimental, are obtained for regular geometries of channels. Otherwise, one has to rely on purely macroscopic observations.

Final remarks

We shall not present in this work the set of field equations for any specific model of soils. However, it should be mentioned that both nonlinear and linear models with fully specified set of material parameters have been developed. The latter models have been also applied in the analysis of acoustic waves for many types of soils (see: [11]). Even though the further theoretical investigations are necessary there exists already a basis for the development of software packages for engineers in which practical problems of deformation and changes of temperature of soils will be coupled with such changes of soil morphology as variations of porosity, saturation, moisture, permeability, and heat conductivity.

References

[1] Bowen R. M. (1976). *Theory of mixtures*, in: Continuum Physics, A. Eringen (ed.), Vol. 3, Academic Press, New York, pp. 2-127.

[2] Bear J. (1988). *Dynamics of Fluids in Porous Media*, Dover Publications, New York.

[3] Bauer E. (1996). *Calibration of a comprehensive hypoplastic model for granular materials*, Soils and Foundations, 36(1), pp. 13-26.

[4] Gassmann F. (1951). *Über die Elastizität poröser Medien*, Vierteljahresschrift der Naturforschenden Gesellschaft in Zürich, 96, pp. 1-23.

[5] Bowen R. M. (1980). *Incompressible porous media models by use of the theory of mixtures*, Int. J. Engng. Sci., 18, pp. 1129-1148.

[6] Bowen R. M. (1982). *Compressible porous media models by use of the theory of mixtures*, Int. J. Engng. Sci., 20(6), pp. 697-763.

[7] Goodman M., Cowin S. (1972). *A continuum theory of granular materials*, Arch. Rat. Mech. Anal., 44, pp. 249-266.

[8] Faria S. H., Hutter K., Kirchner N., Wang Y. (2009). *Continuum description of granular materials*, Springer, Heidelberg.

[9] Wilmanski K. (1996). *Porous media at finite strains. The new model with the balance equation for porosity*, Arch. Mech., 48(4), pp. 591-628.

[10] Wilmanski K. (2008). *Continuum Thermodynamics*, World Scientific, Singapore.

[11] Albers B. (2009). *Modeling and Numerical Analysis of Wave Propagation in Saturated and Partially Saturated Porous Media*, Shaker Verlag, Aachen.

[12] Wilmanski K. (2004). *On Microstructural Tests for Poroelastic Materials and Corresponding Gassmann-type Relations*, Geotechnique, 54(9), pp. 593-603.

[13] Biot M.A., Willis D.G. (1957). *The Elastic Coefficients of the Theory of Consolidation*, J.Appl. Mech., 24, pp. 594-601.

[14] Helmig R. (1997). *Multiphase Flow and Transport Processes in the Subsurface*, Springer, Heidelberg.

[15] Brooks R.H., Corey A.T. (1964). *Hydraulic properties of porous media*. In: Hydrology Papers, vol. 3. Colorado State University, Fort Collins.

[16] Van Genuchten M.T. (1980). *A closed-form equation for predicting the hydraulic conductivity of unsaturated soils*, Soil Sci. Soc. Am. J., vol. 44, pp. 892-898.

[17] Epstein N. (1989). *On tortuosity and the tortuosity factor in flow and diffusion through porous media*, Chem. Eng. Sci., 44, pp. 777-779.

[18] Kozeny J. (1927). *Über kapillare Leitung des Wassers im Boden (Aufstieg, Versickerung und Anwendung auf die Bewässerung)*, Sber. Akad. Wiss. Wien, 136, (Abt. IIa), pp. 271-306.

[19] Biot M. A. (1956). *Theory of propagation of elastic waves in a fluid saturated porous solid, I. Low frequency range*, J. Acoust. Soc. Am., 28(2), pp. 168-178.

[20] Wilmanski K. (2005). *Tortuosity and Objective Relative Accelerations in the Theory of Porous Materials*, Proc. R. Soc. A, 461, pp. 1533-1561.

[21] Wesselingh J. A., Krishna R. (2000). *Mass Transfer in Multicomponent Mixtures*, Delft University Press.