

ADVANCES
IN THE MECHANICS
OF
INHOMOGENEOUS
MEDIA

EDITED BY:

CZESŁAW WOŹNIAK
MIECZYŚŁAW KUCZMA
ROMUALD ŚWITKA
KRZYSZTOF WILMAŃSKI

ZIELONA GÓRA 2010

CHAPTER 9.

DIFFUSION AND HEAT CONDUCTION IN NONLINEAR THERMOPOROELASTIC MEDIA

1. INTRODUCTION

Modeling of poroelastic materials is usually limited to the range of small deformations as the main field of applications of such models is the propagation of acoustic waves. However, many properties of these linear models cannot be explained without referring to nonlinear effects. A typical example is furnished by the celebrated Biot model, Biot (1956)₁ which possesses four important properties seemingly violating principles of continuum thermodynamics:

- coupling of partial stresses by contributions of volume changes of both phases which seems to be forbidden by the second law of thermodynamics (compare: simple mixtures of fluids),
- linear dependence of momentum sources on relative accelerations which seems to violate the principle of objectivity,
- dependence of permeability (resistance to relative motions of components) on the frequency of monochromatic waves which is inconsistent with the linearity and with the temporal form of equations of motion,
- apparent lack of porosity changes which follows from the linearity of constitutive laws.

The first property was analyzed in the paper Wilmanski (2002) and it has been shown that a nonlinear model in which one accounts for a dependence on the porosity gradient yields indeed this coupling in the linear limit. In this sense, the Biot model does not violate the second law.

The linear contribution of relative accelerations is contradicting the principle of objectivity (compare: Wilmanski (2001)). However, a nonlinear correction in the difference of accelerations yields an objective quantity which may appear as a constitutive variable (see: Wilmanski (2005)) and which yields in the linear limit the Biot contribution.

The third flaw of the model results from a misinterpretation of the Biot paper, Biot (1956)₂. In this work M. A. Biot has shown on examples of a few simple flows that the viscosity of real fluids saturating porous materials yields a dependence of permeability on the frequency of acoustic waves. This property shown in a spectral representation of the model (i.e. for independent variables (ω, \mathbf{x}) , where ω - frequency, \mathbf{x} - spatial coordinate) has been taken over by some researchers in a mathematically nonsensical manner (see remarks in Wilmanski (2006)). This flaw can be easily eliminated by an assumption on a hereditary character of the momentum source. However, such a correction yields usually also a nonlinearity of the model.

A proper exploitation of Gedankenexperiments proposed by Gassmann, Gassmann (1951) and then used by Biot and Willis, Biot, Willis (1957) shows that porosity does change in the original Biot model. These changes are different from those presented by Gassmann as they contain, in contrast to the Gassmann relation, both equilibrium and nonequilibrium contributions. However, this point was never properly exposed in the works on linear acoustics. An equilibrium part is proportional to volume changes of the skeleton and the nonequilibrium part is proportional to the difference of volume changes of both components. The latter is, in turn, proportional to the so-called increment of fluid content.

The above mentioned four effects are all related to a macroscopic modeling of complex morphology of a porous material. Even in the case of fully saturated, two-component systems a number of microstructural parameters desired to describe flows in such systems may be quite large. In Figure 1, we show schematically some of these morphological properties which should be reflected in a macroscopic model. We indicate a flow through a chosen channel of a given curvature (part 1)) which may be replaced by a flow through the channel of the same length but a much more complicated shape (part 2)) or by two channels (part 3)) whose average areas of cross-section are together the same as the cross-section in part 1): all three examples are assumed to yield the same local porosity and the same local tortuosity of the system. The latter is understood as a fraction of the length of the channel to the distance between points A and B. It is quite clear that, in spite of the same values of these microstructural parameters: porosity and tortuosity the flows in all three cases are different. This results from local variations of the microscopic relative velocity (a difference of the velocity of true fluid and true skeleton) not accounted for in an average macroscopic relative velocity.

Fig. 1: Three different flows (1)

The true geometrical function $\chi_x(\mathbf{Z}, t)$, vanishes to zero when it belongs to the skeleton (\mathbf{X}, t) , the so-called skeleton. It describes a certain true velocity field of the body in the porous medium. A collection of local velocity fields (\mathbf{X}, t) , is the microscopic relative velocity. The volume averages of the microscopic relative velocity and porosity is defined as

$$n(\mathbf{X}, t)$$

It is clear that the function χ_x

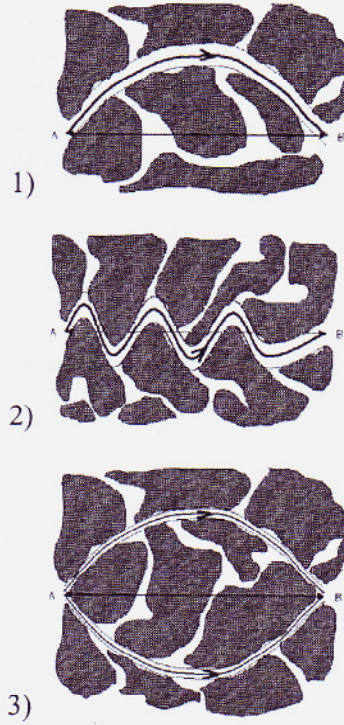


Fig. 1: Three different microstructures of porous material, illustrating three different flows (bold lines with the arrow) in channels of the same tortuosity.

The true geometry of porous materials is fully described by a characteristic function $\chi_X(\mathbf{Z}, t)$, which is equal to one when the point \mathbf{Z} belongs to the fluid and zero when it belongs to the skeleton. The point \mathbf{Z} is an element of the set $REV(\mathbf{X}, t)$, the so-called representative elementary volume, which physically describes a certain true neighborhood of the point \mathbf{X} of the reference configuration \mathcal{B}_0 of the body in the sense of microstructure. It means that a body is understood as a collection of local Cartesian products $\{\mathbf{X}\} \times REV(\mathbf{X}, t)$, $\mathbf{X} \in \mathcal{B}_0$ where $REV(\mathbf{X}, t)$, is the microstructure at the point $\mathbf{X} \in \mathcal{B}_0$. The macroscopic fields are volume averages of true quantities defined on each $REV(\mathbf{X}, t)$. For instance, the porosity is defined by the volume integrat

$$n(\mathbf{X}, t) = \frac{1}{vol REV(\mathbf{X}, t)} \int_{REV(\mathbf{X}, t)} \chi_X(\mathbf{Z}, t) dV_{\mathbf{Z}}, \quad 0 \leq n \leq 1. \quad (1)$$

It is clear that the porosity $n(\mathbf{X}, t)$ is carrying some very rough information about the function $\chi_X(\mathbf{Z}, t): REV(\mathbf{X}, t) \rightarrow \{0, 1\}$. However, such important properties as

a difference between two systems whose porosity is the same but one of them contains a few large cavities in the microstructure $REV(\mathbf{X}, t)$ while the other many small cavities are not described by the porosity alone. The above mentioned tortuosity should be also defined by its relation to the characteristic function χ . Surface porosity of boundaries, important for the formulation of boundary conditions is also not yet defined in this way. One could multiply examples of microstructural variables which are needed for a proper formulation of a macroscopic model of porous materials. This mathematical question seems to be still open. In this work we present an *ad hoc* method of construction of such a model common for this part of continuum thermodynamics.

We exploit thermodynamic conditions imposed on a nonlinear poroelastic model. We account for the dependence on objective relative accelerations and on the porosity gradient. The former is sometimes attributed to tortuosity effects; even though this interpretation seems to be doubtful we show that it describes microscopic fluctuations of relative velocities. The latter, a local nonhomogeneity of the material, allows to introduce couplings of partial stresses mentioned above. Otherwise, as it was demonstrated in earlier papers (e.g. Wilmanski (1998)₂, Wilmanski (2004)₂), the so-called simple mixture model of porous materials follows in which static couplings of components are absent.

We formulate the model in the Lagrangian description with respect to the skeleton which is appropriate for nonlinear modeling. Simultaneously, we construct the model which is almost linear with respect to vectorial constitutive variables, i.e. we assume the linearity of vectorial constitutive laws with respect to the diffusion velocity, temperature gradient, gradient of porosity and relative acceleration. Scalar and tensorial quantities contain also a dependence on some quadratic terms. This is justified by physical expectations.

2. LAGRANGIAN DESCRIPTION OF SATURATED POROUS MEDIA

We begin with the construction of general balance equations for the two-component model. A general case of many components has been considered, for instance, in the paper Wilmanski (2003) but the case of two components is more instructive due to its simplicity. In addition, a multicomponent model must be extended to cover such effects as capillary forces which has not been done yet for nonlinear processes.

We need some basic notions of geometry and kinematics of two-component immiscible mixtures.

The skeleton (solid component of an immiscible mixture) of the porous material is treated as a confinement for the motion of the fluid. It means that a configuration \mathcal{B}_0 , say at the instant of time $t = 0$, is chosen as a reference for both the solid and fluid components. The function of motion of the skeleton

is assumed to be
 $\mathbf{f}^S(\mathbf{X}, t = 0) = \mathbf{X}$
 and deformation

$$\mathbf{x}'^S =$$

The motion of the
 tion of its motion
 velocity of the f

By means of the
 rial domains. For
 configuration \mathcal{Q}

current configuration
 have (compare V

Obviously, the
 $\mathbf{v}^F(\mathbf{x}, t)$, $\mathbf{x} \in \mathcal{Q}$
 kinematics of \mathcal{Q}
 by the relation

Clearly, this rel
 rian description.

For the p
 well a quanti
 $\partial(\mathbf{x}'^F - \mathbf{x}'^S)/\partial t$
 tions of the fra
 cit additive con
 $\mathbf{O}(t)$. However
 z the following

$$\mathbf{x} = \mathbf{f}^S(\mathbf{X}, t), \quad \mathbf{X} \in \mathcal{B}_0, \quad (2)$$

is assumed to be twice differentiable with respect to both arguments. Certainly, $\mathbf{f}^S(\mathbf{X}, t=0) = \mathbf{X}$. Then the velocity of the skeleton, acceleration of the skeleton and deformation gradient are defined in the classical manner

$$\mathbf{x}'^S = \frac{\partial \mathbf{f}^S}{\partial t}(\mathbf{X}, t), \quad \mathbf{x}''^S = \frac{\partial^2 \mathbf{f}^S}{\partial t^2}(\mathbf{X}, t), \quad \mathbf{F}^S = \text{Grad} \mathbf{f}^S(\mathbf{X}, t). \quad (3)$$

The motion of the fluid follows by the transformation of the Euclidean description of its motion to the reference configuration of the skeleton. In particular, the velocity of the fluid has the form

$$\mathbf{x}'^F = \mathbf{v}^F(\mathbf{f}^S(\mathbf{X}, t), t) \quad (4)$$

By means of these velocity fields we can define the Lagrangian image of material domains. For the skeleton, these are, obviously, subsets \mathcal{P}^S of the reference configuration \mathcal{B}_0 . For the fluid, we have to map its material domains from their current configuration \mathcal{P}_t^F to the reference configuration contained in \mathcal{B}_0 . We have (compare Wilmanski (1995))

$$\mathcal{P}^F(t) = \{ \mathbf{X} \in \mathcal{B}_0 \mid \mathbf{f}^S(\mathbf{X}, t) \in \mathcal{P}_t^F \}. \quad (5)$$

Obviously, the kinematics of \mathcal{P}_t^F is given by the field $\mathbf{v}^F(\mathbf{x}, t)$, $\mathbf{x} \in \mathcal{B}_t = \{ \mathbf{x} \mid \mathbf{x} = \mathbf{f}^S(\mathbf{X}, t), \mathbf{X} \in \mathcal{B}_0 \}$, while it can be shown that the kinematics of $\mathcal{P}^F(t)$ is given by the so-called Lagrangian velocity \mathbf{X}'^F defined by the relation

$$\mathbf{X}'^F = \mathbf{F}^{S-1}(\mathbf{x}'^F - \mathbf{x}'^S) = \mathbf{X}'^F(\mathbf{X}, t). \quad (6)$$

Clearly, this relative velocity corresponds to the diffusion velocity in the Eulerian description.

For the purpose of construction of objective constitutive laws we need as well a quantity which is related to the difference of accelerations $\partial(\mathbf{x}'^F - \mathbf{x}'^S)/\partial t$. Such a difference is not objective, i.e. by time-dependent rotations of the frame: $\mathbf{x}^* = \mathbf{O}(t)\mathbf{x}$, $\mathbf{O}^T = \mathbf{O}^{-1}$, its transformation contains explicit additive contributions depending only on the relative motion of the frames $\mathbf{O}(t)$. However, it can be shown, Wilmanski (2005), that for an arbitrary scalar z the following quantity

$$\mathbf{a}_r = \frac{\partial}{\partial t}(\mathbf{x}'^F - \mathbf{x}'^S) - (1-z)\mathbf{X}'^F \cdot \text{Grad } \mathbf{x}'^F - z\mathbf{X}'^F \cdot \text{Grad } \mathbf{x}'^S, \quad (7)$$

is objective, i.e. $\mathbf{a}_r^* = \mathbf{O}\mathbf{a}_r$. It shall be called a Lagrangian acceleration and it shall be used in the constitutive theory.

The above described Lagrangian kinematics yields a peculiar structure of balance equations. We quote them here without details of the derivation which can be found elsewhere (e.g. Wilmanski (1998), (2008))

- mass balance

$$\begin{aligned} R^S &:= \frac{\partial \rho^S}{\partial t} - \hat{\rho}^S = 0, \\ R^F &:= \frac{\partial \rho^F}{\partial t} + \text{Div} \rho^F \mathbf{X}'^F - \hat{\rho}^F = 0, \\ \hat{\rho}^S + \hat{\rho}^F &= 0, \end{aligned} \quad (8)$$

- momentum balance

$$\begin{aligned} \mathbf{M}^S &:= \frac{\partial \rho^S \mathbf{x}'^S}{\partial t} - \text{Div} \mathbf{P}^S - \hat{\mathbf{p}}^S - \rho^S \mathbf{b}^S = 0, \\ \mathbf{M}^F &:= \frac{\partial \rho^F \mathbf{x}'^F}{\partial t} + \text{Div} \rho^F \mathbf{x}'^F \otimes \mathbf{x}'^F - \text{Div} \mathbf{P}^F - \hat{\mathbf{p}}^F - \rho^F \mathbf{b}^F = 0, \\ \hat{\mathbf{p}}^S + \hat{\mathbf{p}}^F &= 0, \end{aligned} \quad (9)$$

- energy balance

$$\begin{aligned} E^S &:= \frac{\partial}{\partial t} \left(\rho^S \varepsilon^S + \frac{1}{2} \rho^S \mathbf{x}'^S \cdot \mathbf{x}'^S \right) + \text{Div} (\mathbf{Q}^S - \mathbf{P}^{ST} \mathbf{x}'^S) - \\ &\quad - \rho^S (\mathbf{b}^S \cdot \mathbf{x}'^S + r^S) - \hat{\varepsilon}^S = 0, \\ E^F &:= \frac{\partial}{\partial t} \left(\rho^F \varepsilon^F + \frac{1}{2} \rho^F \mathbf{x}'^F \cdot \mathbf{x}'^F \right) + \text{Div} \left[(\rho^F \varepsilon^F + \frac{1}{2} \rho^F \mathbf{x}'^F \cdot \mathbf{x}'^F) \mathbf{X}'^F + \right. \\ &\quad \left. + (\mathbf{Q}^F - \mathbf{P}^{FT} \mathbf{x}'^F) \right] - \rho^F (\mathbf{b}^F \cdot \mathbf{x}'^F + r^F) - \hat{\varepsilon}^F = 0, \quad \hat{\varepsilon}^S + \hat{\varepsilon}^F = 0, \end{aligned} \quad (10)$$

- porosity balance

$$D := \frac{\partial \Delta_n}{\partial t} + \text{Div} \mathbf{J} - \hat{n} = 0. \quad (11)$$

We use here the reference volume

describe the material in the domain Ω in the reference configuration among so ρ^{SR}, ρ^{FR} . These

Mass source terms (e.g. due to the skeleton and the fluid) which means that for the fluid component

Lagrangian description \mathcal{P}_i^F , material points of the skeleton (the solid) to remain unchanged

Momentum balance of both $\mathbf{x}'^S, \mathbf{x}'^F$ of both

(\mathbf{X}, t) . The non-linear stresses. The interaction between components, diffusion, a gradient of the skeleton. The skeleton of the second in an arbitrary manner work we assume forces, $\rho^S \mathbf{b}^S, \rho^F \mathbf{b}^F$

Partial energy balance describe local energy for mixtures with such processes. The law will be used to show further. C

We use here the following notation. Mass densities, ρ^S, ρ^F refer to the unit reference volume of the skeleton in its configuration \mathcal{B}_0 , i.e. the integrals

$$\int_{\mathcal{P}} \rho^S dV, \quad \int_{\mathcal{P}} \rho^F dV, \quad (12)$$

describe the mass of the skeleton and the mass of the fluid which are contained in the domain $\mathcal{P} \subset \mathcal{B}_0$ at the instant of time t . This point yields often a confusion among soil specialists who rather use the so-called true mass densities, ρ^{SR}, ρ^{FR} . These will not appear in this paper.

Mass sources, $\hat{\rho}^S, \hat{\rho}^F$ describe the exchange of mass between components (e.g. due to freezing or melting) per unit mass of the reference volume of the skeleton and per unit time. We shall not discuss such problems in this work which means that we neglect these contributions. Partial mass balance equation for the fluid contains the convective term, $\rho^F \mathbf{X}'^F$ which is characteristic for the Lagrangian description. It reflects the fact that projections $\mathcal{P}^F(t)$ of domains \mathcal{P}_t^F , material for the fluid component, on the reference configuration of the skeleton (the so-called pull-back) move with the velocity field \mathbf{X}'^F rather than to remain unchanged as material domains, \mathcal{P}^S , of the skeleton.

Momentum balance equations (9) contain fields of the partial velocities, $\mathbf{x}'^S, \mathbf{x}'^F$ of both components as functions of Lagrangian coordinates and time, (\mathbf{X}, t) . The nonconvective fluxes, $\mathbf{P}^S, \mathbf{P}^F$ are called partial Piola-Kirchhoff stresses. The momentum sources, $\hat{\mathbf{p}}^S, \hat{\mathbf{p}}^F$ describe an exchange of momentum between components. We see further that, in principle, it may be caused by the diffusion, a gradient of porosity, a gradient of temperature and the relative acceleration. The structure of these contributions will be the subject of the exploitation of the second law of thermodynamics. These equations may be represented in an arbitrary non-inertial frame of reference. However, for the purpose of this work we assume the frame to be inertial and, for simplicity, we leave out body forces, $\rho^S \mathbf{b}^S, \rho^F \mathbf{b}^F$ entirely.

Partial energy balance equations (10) contain source terms, $\hat{\mathcal{E}}^S, \hat{\mathcal{E}}^F$. They describe local exchange of energy between components which is characteristic for mixtures with different temperatures of components. We do not consider such processes in this work. For this reason, only the bulk energy conservation law will be used. This follows by the addition of partial equations which we show further. Obviously, $\mathcal{E}^S, \mathcal{E}^F$ are specific internal energies of components

and $\mathbf{Q}^S, \mathbf{Q}^F$ are nonconvective parts of the partial energy fluxes. The terms r^S, r^F describe energy radiation and, similarly, to body forces, we leave them out.

Finally, the porosity balance equation (11) describes the balance of the nonequilibrium part of the porosity

$$\Delta_n = n - n_E, \quad (13)$$

where n is the current porosity and n_E is the constitutive part of the current porosity which is due to changes of equilibrium variables. We return later to the discussion of this representation. The flux of porosity, \mathbf{J} may be caused by diffusion as it is the case in the linear Biot model or by some other processes. We specify them using the second law of thermodynamics. The right-hand side \hat{n} describes the source of porosity which yields a relaxation of porosity changes and it may be caused by some irreversible processes such as damage (formation of microcracks) or melting and freezing.

By means of the above partial balance equations we may easily formulate the bulk conservation laws. Even though we use only the bulk conservation of energy we demonstrate all these equations in order to interpret certain contributions to the bulk energy and the bulk energy flux.

The sum of mass balance equations (8) yields the bulk equation of conservation of mass

$$\frac{\partial \rho}{\partial t} + \text{Div} \rho \dot{\mathbf{X}} = 0, \quad \rho = \rho^S + \rho^F, \quad \rho \dot{\mathbf{X}} = \rho^F \mathbf{X}'^F, \quad (14)$$

where ρ is the bulk mass density and $\dot{\mathbf{X}}$ denotes the bulk Lagrangian velocity. It appears because the motion is referred to the motion of the skeleton. This is different from the classical theory of miscible mixtures where the bulk motion is identical with the so-called barycentric motion. We return to this point after the momentum balance.

The sum of momentum balance equations (9) yields the bulk equation of momentum conservation

$$\frac{\partial \rho \dot{\mathbf{x}}}{\partial t} + \text{Div}(\rho \dot{\mathbf{x}} \otimes \dot{\mathbf{X}} - \mathbf{P}) = 0, \quad \rho \dot{\mathbf{x}} = \rho^S \mathbf{x}'^S + \rho^F \mathbf{x}'^F, \quad (15)$$

and

$$\rho \dot{\mathbf{x}} \otimes \dot{\mathbf{X}} - \mathbf{P} = \rho^F \mathbf{x}'^F \otimes \mathbf{X}'^F - \mathbf{P}^S - \mathbf{P}^F,$$

i.e.

$$\mathbf{P} = \mathbf{P}^S + \mathbf{P}^F$$

In the d
lowing identiti

$$\dot{\mathbf{x}} = \mathbf{x}'^S + \mathbf{F}^S \dot{\mathbf{X}}$$

The bulk velo
ory of mixture
then given by

$$\mathbf{u}^S = -\mathbf{F}^S \dot{\mathbf{X}}$$

Hence, in this
replaces these

It remain
body forces ar

$$\frac{\partial}{\partial t}(\rho \epsilon^b + 1/2 \rho$$

where the rela
 $\rho \epsilon^b + 1/2 \rho$

$$\Rightarrow \rho \epsilon^b = \rho^S$$

and
 $(\rho \epsilon^b + 1/2 \rho$

$$= \mathbf{Q}^S - \mathbf{P}^{ST} \mathbf{x}$$

$$\Rightarrow \mathbf{Q}^b = \mathbf{Q}_I^b$$

whereas the in

$$\mathbf{Q}_I^b = \mathbf{Q}^S + \mathbf{Q}$$

$$\mathbf{P} = \mathbf{P}^S + \mathbf{P}^F - \frac{\rho^S \rho^F}{\rho} \mathbf{F}^S \mathbf{X}'^F \otimes \mathbf{X}'^F \equiv \mathbf{P}^S + \mathbf{P}^F - \rho \frac{\rho^S}{\rho^F} \mathbf{F}^S \dot{\mathbf{X}} \otimes \dot{\mathbf{X}}. \quad (16)$$

In the derivation of the last relation we have used the first two of the following identities

$$\dot{\mathbf{x}} = \mathbf{x}'^S + \mathbf{F}^S \dot{\mathbf{X}}, \quad \dot{\mathbf{x}} = \mathbf{x}'^F - \frac{\rho^S}{\rho^F} \mathbf{F}^S \dot{\mathbf{X}}, \quad \dot{\mathbf{X}} - \mathbf{X}'^F = -\frac{\rho^S}{\rho} \mathbf{X}'^F = -\frac{\rho^S}{\rho^F} \dot{\mathbf{X}}. \quad (17)$$

The bulk velocity $\dot{\mathbf{x}}$ corresponds to the barycentric velocity of the classical theory of mixtures. The diffusion velocities $\mathbf{u}^S = \mathbf{x}'^S - \dot{\mathbf{x}}$ and $\mathbf{u}^F = \mathbf{x}'^F - \dot{\mathbf{x}}$ are then given by the above identities. Namely

$$\mathbf{u}^S = -\mathbf{F}^S \dot{\mathbf{X}} = -\frac{\rho^F}{\rho} \mathbf{F}^S \mathbf{X}'^F, \quad \mathbf{u}^F = \frac{\rho^S}{\rho^F} \mathbf{F}^S \dot{\mathbf{X}} = \frac{\rho^S}{\rho} \mathbf{F}^S \mathbf{X}'^F. \quad (18)$$

Hence, in this two-component system either of the two variables, $\dot{\mathbf{X}}$ or \mathbf{X}'^F , replaces these relative velocities. We shall not use them any further.

It remains to formulate the bulk energy conservation law. If we neglect body forces and radiations it must have the following form

$$\frac{\partial}{\partial t} (\rho \varepsilon^b + 1/2 \rho \dot{\mathbf{x}} \cdot \dot{\mathbf{x}}) + \text{Div} [(\rho \varepsilon^b + 1/2 \rho \dot{\mathbf{x}} \cdot \dot{\mathbf{x}}) \dot{\mathbf{X}} + \mathbf{Q}^b - \mathbf{P}^T \dot{\mathbf{x}}] = 0, \quad (19)$$

where the relations (10) added to each other yield the following identifications

$$\begin{aligned} \rho \varepsilon^b + 1/2 \rho \dot{\mathbf{x}} \cdot \dot{\mathbf{x}} &= \rho^S \varepsilon^S + 1/2 \rho^S \mathbf{x}'^S \mathbf{x}'^S + \rho^F \varepsilon^F + 1/2 \rho^F \mathbf{x}'^F \mathbf{x}'^F \Rightarrow \\ \Rightarrow \rho \varepsilon^b &= \rho^S \varepsilon^S + \rho^F \varepsilon^F + \frac{1}{2} \frac{\rho^S \rho^F}{\rho} \mathbf{X}'^F \cdot \mathbf{C}^S \mathbf{X}'^F, \quad \mathbf{C}^S = \mathbf{F}^{ST} \mathbf{F}^S, \end{aligned} \quad (20)$$

and

$$\begin{aligned} &(\rho \varepsilon^b + 1/2 \rho \dot{\mathbf{x}} \cdot \dot{\mathbf{x}}) \dot{\mathbf{X}} + \mathbf{Q}^b - \mathbf{P}^T \dot{\mathbf{x}} = \\ &= \mathbf{Q}^S - \mathbf{P}^{ST} \mathbf{x}'^S + (\rho^F \varepsilon^F + 1/2 \rho^F \mathbf{x}'^F \cdot \mathbf{x}'^F) \mathbf{X}'^F + \mathbf{Q}^F - \mathbf{P}^{FT} \mathbf{x}'^F \Rightarrow \\ &\Rightarrow \mathbf{Q}^b = \mathbf{Q}_I^b + \frac{1}{2} \frac{\rho^S \rho^F}{\rho} (\rho^S - \rho^F) (\mathbf{X}'^F \cdot \mathbf{C}^S \mathbf{X}'^F) \mathbf{X}'^F, \end{aligned} \quad (21)$$

whereas the intrinsic part \mathbf{Q}_I^b of the energy flux is given by the relation

$$\mathbf{Q}_I^b = \mathbf{Q}^S + \mathbf{Q}^F + \frac{1}{\rho} (\rho^F \mathbf{P}^{ST} - \rho^S \mathbf{P}^{FT}) \mathbf{F}^S \mathbf{X}'^F + \frac{\rho^S \rho^F}{\rho} (\varepsilon^F - \varepsilon^S) \mathbf{X}'^F. \quad (22)$$

There is an alternative to the above definitions of quantities appearing in the full energy balance. Namely, instead of the conservation law of the bulk energy $\rho\epsilon^b + 1/2\rho\dot{\mathbf{x}} \cdot \dot{\mathbf{x}}$, we can use the balance equation for the internal energy defined in the following way

$$\rho\epsilon = \rho^S \epsilon^S + \rho^F \epsilon^F, \quad (23)$$

which is, of course, the intrinsic part of the bulk energy density $\rho\epsilon^b$ (compare (20)). Such a balance equation leaves exposed partial stresses which is convenient in applications as these stresses and not the bulk stress \mathbf{P} appear in the governing equations of the model. Addition of partial equations (10) and application of balance laws for partial mass and momentum yield

$$E := \frac{\partial \rho\epsilon}{\partial t} + \text{Div}(\rho\epsilon \dot{\mathbf{X}} + \mathbf{Q}) - \mathbf{P}^S \cdot \text{Grad} \mathbf{x}'^S - \mathbf{P}^F \cdot \text{Grad} \mathbf{x}'^F - (\mathbf{F}^{ST} \hat{\mathbf{p}}) \mathbf{X}'^F = 0, \quad (24)$$

where we have already neglected mass sources as they do not appear further in this work and other quantities are defined by the relations

$$\rho\epsilon = \rho^S \epsilon^S + \rho^F \epsilon^F, \quad \mathbf{Q} = \mathbf{Q}^S + \mathbf{Q}^F + \frac{\rho^S \rho^F}{\rho} (\epsilon^F - \epsilon^S) \mathbf{X}'^F. \quad (25)$$

The balance equation (24) shall be used in the evaluation of the second law of thermodynamics.

In addition to the above natural balance laws it is convenient to consider a kinematical condition for the existence of the function of motion of the skeleton, \mathbf{f}^S , as one of the equations. This condition consists of two parts (integrability conditions for \mathbf{x}'^S and \mathbf{F}^S)

$$\frac{\partial \mathbf{F}^S}{\partial t} = \text{Grad} \mathbf{x}'^S, \quad \frac{\partial F_{kK}^S}{\partial X_L} = \frac{\partial F_{kL}^S}{\partial X_K}, \quad (26)$$

where, for clarity, the second relation was written in the Cartesian frame: $\mathbf{F}^S = F_{kK}^S \mathbf{e}_k \otimes \mathbf{e}_K$, $\mathbf{e}_k, \mathbf{e}_K$ being base vectors in current and, correspondingly, reference configurations. The second integrability condition is usually directly incorporated in constitutive assumptions. However, it is convenient to consider the first one as an additional balance relation of the following divergent form

$$\mathbf{F} := \frac{\partial \mathbf{F}^S}{\partial t} - \text{Div}(\mathbf{x}'^S \otimes \mathbf{1}) = 0, \quad (27)$$

where $\mathbf{1}$ denotes the identity tensor, $\mathbf{1} = \delta_{KL} \mathbf{e}_K \otimes \mathbf{e}_L$.

3. ENTROPY

The thermodynamic balance is derived from the following fundamental principle:

It means that the constitutive equations of the structural variables must be such that the balance laws of the quantities whose need is not present a typical material. However, the balance laws are incorporated in the constitutive equations. Field equations of the balance laws must be satisfied by the constitutive relations.

Such relations are used in the work, we consider the constitutive relations and the balance laws.

$$\mathcal{R} = \{\rho^S, \epsilon^S, \mathbf{f}^S, \mathbf{F}^S, \mathbf{Q}^S, \mathbf{P}^S, \mathbf{Q}^F, \mathbf{P}^F, \mathbf{F}^{ST}, \mathbf{p}\}$$

where the relations between the partial mass and the partial energy are both partial mass and the partial energy between components of the deformation gradient. The deformation gradient is the deformation gradient (diffusion), Δ_n is the character of the process and \mathbf{a}_r for relation of Biot's model of certain coupled fields. The present model is based on the present model of Biot's model (2002) and the constitutive relations.

where $\mathbf{1}$ denotes the Cartesian metric of Lagrangian coordinates, i.e. $\mathbf{1} = \delta_{KL} \mathbf{e}_K \otimes \mathbf{e}_L$.

3. ENTROPY INEQUALITY

The thermoporoelastic model which we consider in this work must specify the following fundamental fields

$$\mathcal{F} = \{\rho^S, \rho^F, \mathbf{x}'^S, \mathbf{x}'^F, \mathbf{F}^S, T, n\} \quad (28)$$

It means that the model is two-component and that it contains only one microstructural variable - porosity, n . Tortuosity or any other microstructural quantities whose necessity were indicated in Introduction do not appear. This is at present a typical situation in construction of macroscopic models of porous materials. However, in an implicit way some additional properties of microstructure are incorporated in the present model and this will be indicated in the sequel. Field equations for the fields (28) which follow from the above presented balance laws must be constructed by means of the closure relations. These are constitutive relations for the constitutive quantities of the model

$$\mathcal{C} = \{\hat{\rho}^S, \mathbf{P}^S, \mathbf{P}^F, \hat{\mathbf{p}}^S, \varepsilon, \mathbf{Q}, \mathbf{J}, \hat{n}\} \quad (29)$$

Such relations may be functional or differential or a mixture of both. In this work, we consider a so-called thermoelastic porous materials for which constitutive relations are functions of the following set of constitutive variables

$$\mathcal{R} = \{\rho^S, \rho^F, \mathbf{F}^S, \mathbf{X}'^F, \Delta_n, T, \mathbf{G}, \mathbf{N}, \mathbf{a}_r\}, \quad \mathbf{G} = \text{Grad } T, \quad \mathbf{N} = \text{Grad } n, \quad (30)$$

where the relative acceleration \mathbf{a}_r is defined by the relation (7). The presence of both partial mass densities is necessary only in the case of mass exchange between components. Otherwise, only the mass density ρ^F would appear in order to describe volume changes of the fluid component. Obviously, \mathbf{F}^S accounts for the deformations of the skeleton, \mathbf{X}'^F for the relative motion of components (diffusion), Δ_n for nonequilibrium changes of porosity, T for nonisothermal character of processes, \mathbf{G} for heat conduction, \mathbf{N} for heterogeneity of porosity, and \mathbf{a}_r for relative acceleration of components. Experience with the derivation of Biot's model shows that the dependence on \mathbf{N} is essential for the admissibility of certain couplings in a linear model (e.g. Wilmanski(2004)₁). Consequently, the present model is a generalization of those described in earlier works Wilmanski (2002) and Wilmanski (2005).

Constitutive relations

$$C = C(\mathcal{R}), \quad (31)$$

contain in addition a parametric dependence on the initial porosity n_0 . In the case of residual partial stresses which may appear in the configuration for $t = 0$, they should also be present in a parametric way in constitutive relations. It is, for instance the case when the fluid component is the gas. Constitutive relations should satisfy a set of principles characteristic for a thermodynamic strategy of constructing continuous models. We shall not present here any details of this strategy referring the reader to monographs on the subject (e.g. Wilmanski (2008)). However, in some details we discuss consequences of the second law of thermodynamics. We proceed to formulate this law in the Lagrangian description of two-component porous materials.

The fundamental assumption of this formulation refers to the existence of partial entropy densities, η^S, η^F , which satisfy balance laws

$$\frac{\partial \rho^S \eta^S}{\partial t} + \text{Div} \mathbf{H}^S = \hat{\eta}^S, \quad \frac{\partial \rho^F \eta^F}{\partial t} + \text{Div}(\rho^F \eta^F \mathbf{X}'^F + \mathbf{H}^F) = \hat{\eta}^F, \quad (32)$$

in which $\mathbf{H}^S, \mathbf{H}^F$ are partial entropy fluxes, and $\hat{\eta}^S, \hat{\eta}^F$ are partial entropy sources. Addition of these relations yields

$$\frac{\partial \rho \eta}{\partial t} + \text{Div}(\rho \eta \dot{\mathbf{X}} + \mathbf{H}) = \hat{\eta}^S + \hat{\eta}^F, \quad (33)$$

where

$$\rho \eta = \rho^S \eta^S + \rho^F \eta^F, \quad \mathbf{H} = \mathbf{H}^S + \mathbf{H}^F + \frac{\rho^S \rho^F}{\rho} (\eta^F - \eta^S) \mathbf{X}'^F. \quad (34)$$

The most essential part of the second law of thermodynamics which we consider in this work is the assumption that the source of entropy $\hat{\eta}^S + \hat{\eta}^F$ is nonnegative for all solutions of field equations. This yields the following entropy inequality

$$\frac{\partial \rho \eta}{\partial t} + \text{Div}(\rho \eta \dot{\mathbf{X}} + \mathbf{H}) \geq 0, \quad \eta = \eta(\mathcal{R}), \quad \mathbf{H} = \mathbf{H}(\mathcal{R}), \quad (35)$$

which must hold for all admissible processes, i.e. solutions of field equations.

Comparison of relations (22) and (34) reveals that even in the case of classical Fourier relations between partial fluxes $\mathbf{H}^S = \mathbf{Q}^S/T$ and $\mathbf{H}^F = \mathbf{Q}^F/T$, it can be hardly expected that a similar relation holds for the bulk

fluxes \mathbf{Q} and \mathbf{H} of fluids and it holds more obvious as due to the contact not possess such other, one can n

$$\mathbf{Q}_I =$$

where

i.e. p^S, p^F are ties. In such a c but the coefficient thalpies) in \mathbf{Q}_I ted only by the chemical potent (i.e. $(\varepsilon^F + p^F)$ which, of course

Even less not only Fourier the second law ly continuity co ently, a measur made for mixtu still not comple

Exploitat constraints on t case of the mo ality

$$\frac{\partial}{\partial t}$$

fluxes \mathbf{Q} and \mathbf{H} . This observation has been made by I. Müller for mixtures of fluids and it holds also for porous materials. For immiscible mixtures it is even more obvious as the intrinsic flux \mathbf{Q}_I is anisotropic with respect to the diffusion due to the contribution of partial stresses (compare (22)) while \mathbf{H} in (34) does not possess such a contribution. In a particular case, when, for some reason or other, one can neglect an influence of shear stresses in the relation (22) it follows

$$\mathbf{Q}_I = \mathbf{Q}^S + \mathbf{Q}^F + \frac{\rho^S \rho^F}{\rho} \left(\left(\varepsilon^F + \frac{p^F}{\rho_i^F} \right) - \left(\varepsilon^S + \frac{p^S}{\rho_i^S} \right) \right) \mathbf{X}'^F, \quad (36)$$

where

$$\begin{aligned} p^S &= -\frac{1}{3} J^{S-1} \text{tr} \mathbf{P}^S \mathbf{F}^{ST}, \quad p^F = -\frac{1}{3} J^{S-1} \text{tr} \mathbf{P}^F \mathbf{F}^{ST}, \\ \rho_i^S &= J^{S-1} \rho^S, \quad \rho_i^F = J^{S-1} \rho^F, \quad J^S \equiv \text{tr} \mathbf{F}^S, \end{aligned} \quad (37)$$

i.e. p^S, p^F are partial pressures, while ρ_i^S, ρ_i^F are current partial mass densities. In such a case, the second part of the intrinsic flux \mathbf{Q}_I is parallel to \mathbf{X}'^F but the coefficients $(\varepsilon^F + p^F/\rho_i^F) - (\varepsilon^S + p^S/\rho_i^S)$ (difference of specific enthalpies) in \mathbf{Q}_I and $\eta^F - \eta^S$ (difference of specific entropies) in \mathbf{H} are not related only by the factor $1/T$. If this were the case then this would mean equal chemical potentials of components

$$(\text{i.e. } (\varepsilon^F + p^F/\rho_i^F - T\eta^F) = (\varepsilon^S + p^S/\rho_i^S - T\eta^S))$$

which, of course, does not hold in general.

Even less one can say about different partial temperatures. In such a case not only Fourier relations do not hold but, in addition, some other components of the second law cannot be formulated in a standard way. This concerns particularly continuity conditions on boundaries between different systems and, consequently, a measurability of partial temperatures. Some progress has been recently made for mixtures of gases, Ruggeri, Simic (2007), but even there the solution is still not complete.

Exploitation of the entropy inequality (35) is based on the elimination of constraints on the space of its solutions by means of Lagrange multipliers. In the case of the model under consideration one has to consider the following inequality

$$\begin{aligned} & \frac{\partial \rho \eta}{\partial t} + \text{Div}(\rho \eta \dot{\mathbf{X}} + \mathbf{H}) - \Lambda^{\rho^S} R^S - \Lambda^{\rho^F} R^F - \\ & - \Lambda^{\nu^S} \cdot \mathbf{M}^S - \Lambda^{\nu^F} \cdot \mathbf{M}^F - \Lambda^\varepsilon E - \Lambda^\eta D - \Lambda^{F^S} \cdot \mathbf{F} \geq 0, \end{aligned} \quad (38)$$

for all fields (28). The multipliers $\Lambda^{\rho^S}, \Lambda^{\rho^F}, \Lambda^{\nu^S}, \Lambda^{\nu^F}, \Lambda^E, \Lambda^n, \Lambda^{F^S}$ are functions of constitutive variables (25). In contrast to the entropy inequality (30), the above inequality must hold for arbitrary fields and not only for solutions of field equations.

4. SIMPLIFIED CONSTITUTIVE RELATIONS

Apart from usual restrictions of material objectivity, we assume additionally that the system is isotropic and that a dependence on vectorial constitutive variables $\mathbf{X}'^F, \mathbf{G}, \mathbf{N}, \mathbf{a}_r$ has the lowest nontrivial order. The same assumption concerns the deviation of porosity from equilibrium, Δ_n . For the purpose of this work we neglect as well the exchange of mass between components $\hat{\rho}^S = 0$. Then the mass density ρ^S does not appear as a constitutive variable and the multiplier $\Lambda^{\rho^S} = 0$.

These assumptions yield the following structure of constitutive relations

- - partial stresses

$$\mathbf{P}^S = \mathbf{P}_0^S(\mathcal{R}_E, \Delta_n) + \frac{1}{2} \sigma^S(\mathcal{R}_E) \mathbf{F}^S \mathbf{X}'^F \otimes \mathbf{X}'^F, \quad \mathcal{R}_E = \{\rho^F, \mathbf{F}^S, T\}, \quad (39)$$

$$\mathbf{P}^F = \mathbf{P}_0^F(\mathcal{R}_E, \Delta_n) + \frac{1}{2} \sigma^F(\mathcal{R}_E) \mathbf{F}^S \mathbf{X}'^F \otimes \mathbf{X}'^F, \quad n_E = n_E(\mathcal{R}_E), \quad (40)$$

- - internal energy and entropy

$$\rho \mathcal{E} = \rho \mathcal{E}_0(\mathcal{R}_E, \Delta_n) + \frac{1}{2} \varepsilon_d(\mathcal{R}_E) (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F), \quad (41)$$

$$\rho \eta = \rho \eta_0(\mathcal{R}_E, \Delta_n) + \frac{1}{2} \eta_d(\mathcal{R}_E) (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F), \quad (42)$$

- - fluxes of energy, entropy and porosity

$$\begin{aligned} \mathbf{Q} &= Q_V \mathbf{X}'^F - K \mathbf{G} + Q_n \mathbf{N} + Q_a \mathbf{F}^{ST} \mathbf{a}_r, \\ \mathbf{H} &= H_V \mathbf{X}'^F + H_T \mathbf{G} + H_n \mathbf{N} + H_a \mathbf{F}^{ST} \mathbf{a}_r, \\ \mathbf{J} &= \Phi \mathbf{X}'^F + J_T \mathbf{G} + J_n \mathbf{N} + J_a \mathbf{F}^{ST} \mathbf{a}_r, \end{aligned} \quad (43)$$

where all coefficients are functions of variables \mathcal{R}_E ,

- momentum source

\mathbf{F}^S

with coefficient

There is a physical argument for a considerable tensorial

Nonlinear

and entropy (ma

partial stress te

connections after

tion for such te

volume averaging

account for rap

may be considere

ty, i.e. by the cu

Such fluctuations

create a kind of

ideal fluids in an

Simultane

\mathbf{G}, \mathbf{N} and \mathbf{a}_r .

vectors. There e

near way on the

Obviously,

ductivity.

Coefficie

and entropy. Ex

of bulk quantiti

butions in \mathbf{X}'^F

not need to go d

by the above co

within this mod

ations based on

when the comp

that certain pro

of momentum s

This concerns, f

manski (2002)).

sion velocity w

tion.

$$\mathbf{F}^{ST} \hat{\mathbf{p}} = \Pi_V \mathbf{X}'^F + \Pi_T \mathbf{G} + \Pi_N \mathbf{N} - \rho_{12}^0 \mathbf{F}^{ST} \mathbf{a}_r, \quad \hat{\mathbf{p}} \equiv \hat{\mathbf{p}}^S, \quad (44)$$

with coefficients dependent again on variables \mathcal{R}_E .

There is an obvious lack of equipresence in these relations. This is related to physical arguments which we proceed to present and, simultaneously, it yields considerable technical simplifications.

Nonlinear contributions of diffusion velocity \mathbf{X}'^F to the internal energy and entropy (material parameters ε_d and η_d) are related to analogous terms in partial stress tensors (material parameters σ^S and σ^F). We shall find their connections after the exploitation of the entropy inequality. The physical motivation for such terms follows from the analysis of microstructure. Namely, the volume averaging shows that the macroscopic momenta $\rho^S \mathbf{x}'^S, \rho^F \mathbf{x}'^F$ do not account for rapid changes of directions of true microscopic velocities. These may be considered to be fluctuations of partial momenta caused by the tortuosity, i.e. by the curvy character of channels in porous materials (see: Introduction). Such fluctuations are one of the reasons for dissipation of acoustic waves and create a kind of geometrical viscosity of the diffusive flow even in the case of ideal fluids in an elastic porous matrix.

Simultaneously, the internal energy and the entropy do not depend on \mathbf{G}, \mathbf{N} and \mathbf{a}_r . They are scalars and as such cannot depend in a linear way on vectors. There exists also no clear argument that they should depend in a nonlinear way on these variables.

Obviously, coefficients K and H_T are related to the usual thermal conductivity.

Coefficients Q_V and H_V are responsible for diffusive fluxes of energy and entropy. Explicit relations (21), (22) and (34) following from the definitions of bulk quantities are compatible with (43) only if we neglect nonlinear contributions in \mathbf{X}'^F and leave out anisotropic effects caused by shear stresses. We do not need to go deeply into these simplifications because the bulk fluxes specified by the above constitutive relations will be not related to their partial counterparts within this model. Such a relation may be important if we consider field equations based on partial energy balance equations and this would be the case when the components had different temperatures. However, one should stress that certain processes in granular materials are related to a nonlinear dependence of momentum source (diffusive force) on the relative velocity of components. This concerns, for instance, the liquefaction and piping of soils, (Wilhelm, Wilmanski (2002)). A model describing such processes must be nonlinear in diffusion velocity which is natural as these velocities are large during the liquefaction.

Coefficients Q_n, Q_a in the flux \mathbf{Q} and H_n, H_a in the flux \mathbf{H} follow from the extension of the model to a constitutive dependence on heterogeneity of porosity and on relative acceleration. We discuss them after the presentation of thermodynamic identities following from the second law.

The structure of the relation for the momentum source is similar. The coefficient Π_v corresponds to the classical diffusion force. The coefficient Π_n relates the source to the heterogeneity of porosity. We shall see that, similarly to the contribution with the coupling coefficient ρ_{12}^0 , this contribution is nondissipative.

Simplification with respect to the dependence on Δ_n has not been yet fully exposed. Dependence on this field does not appear in vectorial fluxes but it is present in the energy contribution ε_0 , entropy contribution η_0 and both partial stresses $\mathbf{P}_0^S, \mathbf{P}_0^F$. We return to this dependence in the next Section.

5. EVALUATION OF ENTROPY INEQUALITY

Substitution of the above discussed constitutive relations in the entropy inequality (38) yields the inequality linear with respect to the following time derivatives

$$\left\{ \frac{\partial \rho^F}{\partial t}, \frac{\partial \mathbf{F}^S}{\partial t}, \frac{\partial \Delta_n}{\partial t}, \frac{\partial \mathbf{x}'^S}{\partial t}, \frac{\partial \mathbf{x}'^F}{\partial t}, \frac{\partial T}{\partial t} \right\}. \quad (45)$$

Time derivatives of the remaining constitutive variables \mathcal{R} i.e. $\partial \mathbf{G} / \partial t, \partial \mathbf{N} / \partial t$, do not enter the inequality due to our simplifying assumptions. \mathbf{G} and \mathbf{N} appear only in fluxes which are not differentiated with respect to time. Simultaneously, the contribution with the relative acceleration \mathbf{a}_r yields additional terms in time derivatives $\partial \mathbf{x}'^S / \partial t, \partial \mathbf{x}'^F / \partial t$.

According to Liu's Theorem (e.g. Wilmanski (2008)), it means that coefficients of these derivatives must vanish. Consequently, we obtain the following set of relations for multipliers

$$\begin{aligned} \frac{\partial \rho^F}{\partial t}: \quad \Lambda^{\rho^F} &= \Lambda_0^{\rho^F} + \frac{1}{2} \left(\frac{\partial \eta_d}{\partial \rho^F} - \Lambda^\varepsilon \frac{\partial \varepsilon_d}{\partial \rho^F} \right) (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F), \\ \Lambda_0^{\rho^F} &:= \frac{\partial \rho \eta_0}{\partial \rho^F} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_0}{\partial \rho^F}, \end{aligned} \quad (46)$$

$$\frac{\partial \mathbf{F}^S}{\partial t}: \quad \Lambda^F =$$

$$\frac{\partial \mathbf{x}'^S}{\partial t}: \quad$$

$$(\rho^S - \rho_{12}^0) \Lambda^{\nu^S} + \text{Div} (H_a \mathbf{F}^S)$$

$$\frac{\partial \mathbf{x}'^F}{\partial t}: \quad$$

$$(\rho^F - \rho_{12}^0) \Lambda^{\nu^F} - \text{Div} (H_a \mathbf{F}^S)$$

$$\frac{\partial T}{\partial t}: \quad \frac{\partial \rho \eta_0}{\partial T}$$

As usually done considered to be

Inspection system and which least a linear hom

pation contains

Hence $\rho \varepsilon_0$ and

both functions n

order in Δ_n).

In addit

$\text{Grad} T, \text{Grad} \rho$

quantities are a

coefficients zero

$$\frac{\partial \mathbf{F}^S}{\partial t} : \Lambda^F = \frac{\partial \rho \eta_0}{\partial \mathbf{F}^S} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_0}{\partial \mathbf{F}^S} + \frac{1}{2} \left(\frac{\partial \eta_d}{\partial \mathbf{F}^S} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_d}{\partial \mathbf{F}^S} \right) (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F), \quad (47)$$

$$\frac{\partial \Delta_n}{\partial t} : \Lambda^n = \frac{\partial \rho \eta_0}{\partial \Delta_n} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_0}{\partial \Delta_n}, \quad (48)$$

$$\begin{aligned} \frac{\partial \mathbf{x}'^S}{\partial t} : \\ (\rho^S - \rho_{12}^0) \Lambda^{\nu^S} + \rho_{12}^0 \Lambda^{\nu^F} + (\eta_d - \Lambda^\varepsilon \varepsilon_d) (\mathbf{F}^S \mathbf{X}'^F) - \rho_{12}^0 \Lambda^\varepsilon (\mathbf{F}^S \mathbf{X}'^F) + \\ + \text{Div} (H_a \mathbf{F}^S) - \Lambda^\varepsilon \text{Div} (Q_a \mathbf{F}^S) - \Lambda^n \text{Div} (J_a \mathbf{F}^S) = 0, \end{aligned} \quad (49)$$

$$\begin{aligned} \frac{\partial \mathbf{x}'^F}{\partial t} : \\ (\rho^F - \rho_{12}^0) \Lambda^{\nu^F} + \rho_{12}^0 \Lambda^{\nu^S} - (\eta_d - \Lambda^\varepsilon \varepsilon_d) (\mathbf{F}^S \mathbf{X}'^F) + \rho_{12}^0 \Lambda^\varepsilon (\mathbf{F}^S \mathbf{X}'^F) - \\ - \text{Div} (H_a \mathbf{F}^S) + \Lambda^\varepsilon \text{Div} (Q_a \mathbf{F}^S) + \Lambda^n \text{Div} (J_a \mathbf{F}^S) = 0, \end{aligned} \quad (50)$$

$$\frac{\partial T}{\partial t} : \frac{\partial \rho \eta_0}{\partial T} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_0}{\partial T} + \frac{1}{2} \left(\frac{\partial \eta_d}{\partial T} - \Lambda^\varepsilon \frac{\partial \varepsilon_d}{\partial T} \right) (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F) = 0. \quad (51)$$

As usually done, we identify Λ^ε in a different way and the relation (51) will be considered to be an additional restriction on constitutive functions.

Inspection of the residual inequality which describes the dissipation of the system and which we present further shows that the multiplier Λ^n must be at least a linear homogeneous function of Δ_n . This is due to the fact that the dissipation contains the contribution $\Lambda^n \Delta_n$ which must be nonnegative for all Δ_n . Hence $\rho \varepsilon_0$ and $\rho \eta_0$ must be at least quadratic with respect to Δ_n . However both functions may also possess contributions independent of Δ_n (i.e. zeroth order in Δ_n).

In addition, the above relations contain terms linear in $\text{Grad} T, \text{Grad} \rho^F, \text{Grad} \mathbf{F}^S$, as well as linear and quadratic in \mathbf{X}'^F . As these quantities are arbitrary, we must impose additional restrictions yielding their coefficients zero. After a straightforward analysis, we obtain

$$J_a = 0, \quad (52)$$

$$\begin{aligned} \frac{\partial H_a}{\partial T} - \Lambda^\varepsilon \frac{\partial Q_a}{\partial T} = 0, \quad \frac{\partial H_a}{\partial \rho^F} - \Lambda^\varepsilon \frac{\partial Q_a}{\partial \rho^F} = 0, \\ \left(\frac{\partial H_a}{\partial F_{IL}^S} - \Lambda^\varepsilon \frac{\partial Q_a}{\partial F_{IL}^S} \right) F_{kK}^S + (H_a - \Lambda^\varepsilon Q_a) \delta_{kl} \delta_{KL} = 0, \end{aligned} \quad (53)$$

$$\rho^S \Lambda^{\nu^S} = -\rho^F \Lambda^{\nu^F} = y \mathbf{F}^S \mathbf{X}'^F, \quad y := \frac{\eta_d - \Lambda^\varepsilon \varepsilon_d - \Lambda^\varepsilon \rho_{12}^0}{\rho^S - \rho_{12}^0 (1 + \rho^S / \rho^F)}. \quad (54)$$

Relations (53) yield integrability conditions which could be satisfied if Λ^ε were a constant. This is not possible. Consequently, we have to require

$$H_a = 0, \quad Q_a = 0. \quad (55)$$

The inequality following from (38) is also linear with respect to the following spatial derivatives

$$\text{Grad } \mathbf{x}'^S, \text{Grad } \mathbf{x}'^F, \text{Grad } \rho^F, \text{Grad } \mathbf{F}^S, \text{Grad } \mathbf{G}, \text{Grad } \mathbf{N}. \quad (56)$$

The contribution of $\text{Grad } \Delta_n$ does not appear among them because it can be written in the following explicit form

$$\begin{aligned} \text{Grad } \Delta_n &= \mathbf{N} - \frac{\partial n_E}{\partial T} \mathbf{G} - \frac{\partial n_E}{\partial \rho^F} \text{Grad } \rho^F - \frac{\partial n_E}{\partial F_{kK}^S} \frac{\partial F_{kK}^S}{\partial X_L} \mathbf{e}_L, \\ \text{Grad } \mathbf{F}^S &\equiv \frac{\partial F_{kK}^S}{\partial X_L} \mathbf{e}_k \otimes \mathbf{e}_K \otimes \mathbf{e}_L, \end{aligned} \quad (57)$$

i.e. except of the first two terms, it is a combination the gradients already listed in (56). We have listed them in the order of the further analysis and, simultaneously, skipped the derivative $\text{Grad } \mathbf{a}$, because it does not contribute to the second law due to the relations (52) and (55). The linearity with respect to (56) yields a set of identities and leaves a residual inequality which is essentially nonlinear. It defines the dissipation in the system and has the following form

$$\begin{aligned} \mathcal{D} := & \left(\frac{\partial H_\nu}{\partial T} - \Lambda^\varepsilon \frac{\partial Q_\nu}{\partial T} + \Pi_\tau \right) \mathbf{X}'^F \cdot \mathbf{G} + \\ & + \left(\frac{\partial H_\tau}{\partial T} + \Lambda^\varepsilon \frac{\partial K}{\partial T} - \Lambda^n \frac{\partial J_\tau}{\partial T} \right) \mathbf{G} \cdot \mathbf{G} + \Lambda^\varepsilon \Pi_\nu \mathbf{X}'^F \cdot \mathbf{X}'^F + \Lambda^n \hat{n} \geq 0. \end{aligned} \quad (58)$$

Hence the state

i.e. the temperature, porosity cause acceleration and common claims be considered to be considered in the first of the microscopic

As already of Δ_n . This ind

$\hat{n} =$

with $\lambda'' < 0$. So ar contributions

Now we

i) The co

$$\Lambda^\varepsilon \mathbf{P}_0^S \mathbf{F}^{ST} + \left(\frac{\partial}{\partial} \right)$$

$$2 \left(\frac{\partial \eta_d}{\partial III} - \right)$$

where

Hence the state of thermodynamical equilibrium defined by $\mathcal{D} = 0$ appears if

$$\mathbf{G}|_E = 0, \quad \mathbf{X}'^F|_E = 0, \quad \hat{n}|_E = 0, \quad (59)$$

i.e. the temperature gradient, the relative motion (diffusion), and the source of porosity cause the deviation from the equilibrium. Consequently, the relative acceleration \mathbf{a}_r has no influence on the dissipation. For this reason, in spite of common claims in the literature concerning Biot's model, the tortuosity cannot be considered to be responsible for the presence of \mathbf{a}_r in the model. As indicated in the first Section of this work, tortuosity has an influence on true flows on the microscopic level which must be dissipative.

As already mentioned the multiplier Λ^n is a linear homogeneous function of Δ_n . This indeed follows from the inequality (58). We can write

$$\hat{n} = -\frac{\Delta_n}{\tau}, \quad \Lambda^n = \lambda^n \Delta_n, \quad \tau = \tau(\mathcal{R}_E), \quad \lambda^n = \lambda^n(\mathcal{R}_E). \quad (60)$$

with $\lambda^n < 0$. Simultaneously, the dissipation inequality indicates (compare linear contributions of Δ_n to the first two terms in dissipation)

$$\frac{\partial \Phi}{\partial T} = 0, \quad \frac{\partial J_n}{\partial T} = 0. \quad (61)$$

Now we return to the coefficients of derivatives (56).

i) The coefficient of $\text{Grad} \mathbf{x}'^S$ yields

$$\Lambda^\epsilon \mathbf{P}_0^S \mathbf{F}^{ST} + \left(\frac{\partial \rho \eta_0}{\partial \mathbf{F}^S} - \Lambda^\epsilon \frac{\partial \rho \epsilon_0}{\partial \mathbf{F}^S} \right) \mathbf{F}^{ST} + (-H_V + \Lambda^\epsilon Q_V + \Lambda^n \Phi + \rho^F \Lambda_0^{\rho^F}) \mathbf{1} = 0, \quad (62)$$

$$\frac{\partial \eta_d}{\partial I} - \Lambda^\epsilon \frac{\partial \epsilon_d}{\partial I} = 0, \quad \frac{\partial \eta_d}{\partial II} - \Lambda^\epsilon \frac{\partial \epsilon_d}{\partial II} = 0, \quad (63)$$

$$2 \left(\frac{\partial \eta_d}{\partial III} - \Lambda^\epsilon \frac{\partial \epsilon_d}{\partial III} \right) III + \rho^F \left(\frac{\partial \eta_d}{\partial \rho^F} - \Lambda^\epsilon \frac{\partial \epsilon_d}{\partial \rho^F} \right) = y \left(\sigma^S + \frac{\rho^S}{\rho^F} \sigma^F \right), \quad (64)$$

$$z = -\frac{1}{2\rho_{12}^0} \frac{\sigma^S \Lambda^\epsilon - y(\sigma^S + \rho^S/\rho^F \sigma^F)}{y(1 + \rho^S/\rho^F) + \Lambda^\epsilon}, \quad (65)$$

where

$$I := \text{tr} \mathbf{C}^S, \quad II := \frac{1}{2} (I^2 - \text{tr} \mathbf{C}^S), \quad III := \det \mathbf{C}^S, \quad \mathbf{C}^S = \mathbf{F}^{ST} \mathbf{F}^S, \quad (66)$$

are main invariants of the Cauchy-Green deformation tensor \mathbf{C}^S . In order to simplify calculations we have already accounted for the relation (47) and eliminated the multiplier Λ^F . Relations (63), (64), follow from the quadratic dependence of identities with respect to Δ_n and the relation (65) results from the quadratic dependence of identities with respect to \mathbf{X}'^F . We have used the assumption on the isotropy with respect to the dependence on the deformation of the skeleton which is described by the deformation tensor \mathbf{C}^S .

ii) Similarly, the coefficient of the derivative $\text{Grad} \mathbf{x}'^F$ yields

$$\Lambda^\varepsilon \mathbf{P}_0^F = - (H_V - \Lambda^\varepsilon Q_V - \rho^F \Lambda^{\rho^F} - \Lambda^n \Phi) \mathbf{F}^{S-T}, \quad (67)$$

$$\frac{\partial \eta_d}{\partial III} - \Lambda^\varepsilon \frac{\partial \varepsilon_d}{\partial III} = 0, \quad (68)$$

$$\frac{1}{2} \Lambda^\varepsilon (\sigma^S + \sigma^F) = -\rho_{12}^0 \left(1 + \frac{\rho^S}{\rho^F} \right) y - \rho^S y - \rho_{12}^0 \Lambda^\varepsilon. \quad (69)$$

Consequently, bearing (63) and (68) in mind,

$$\frac{\partial \eta_d}{\partial \mathbf{F}^S} - \Lambda^\varepsilon \frac{\partial \varepsilon_d}{\partial \mathbf{F}^S} = 0. \quad (70)$$

It is easy to check that these results are not influenced by the presence of \mathbf{N} as a constitutive variable. Consequently, they do not differ from those derived in the paper Wilmanski (2005).

iii) We proceed to the derivatives $\text{Grad} \rho^F$ and $\text{Grad} \mathbf{F}^S$. We have

$$\begin{aligned} & \left(\frac{\partial H_V}{\partial \rho^F} - \Lambda^\varepsilon \frac{\partial Q_V}{\partial \rho^F} - \Lambda^n \frac{\partial \Phi}{\partial \rho^F} - \Lambda^{\rho^F} \right) \mathbf{X}'^F + \\ & + y \frac{\partial \mathbf{P}^{ST}}{\partial \rho^F} \mathbf{F}^S \mathbf{X}'^F - y \frac{\rho^F}{\rho^S} \frac{\partial \mathbf{P}^{FT}}{\partial \rho^F} \mathbf{F}^S \mathbf{X}'^F = 0, \end{aligned} \quad (71)$$

$$\begin{aligned} & \text{sym} \left\{ \left(\frac{\partial H_V}{\partial \mathbf{F}^S} - \Lambda^\varepsilon \frac{\partial Q_V}{\partial \mathbf{F}^S} - \Lambda^n \frac{\partial \Phi}{\partial \mathbf{F}^S} \right) \otimes \mathbf{X}'^F + \rho^F \Lambda^{\rho^F} \mathbf{F}^{S-T} \otimes \mathbf{X}'^F + \right. \\ & \left. + \gamma \Xi^S + \gamma \Xi^F \right\} - \text{sym} \left\{ (H_V - \Lambda^\varepsilon Q_V - \Lambda^n \Phi) \mathbf{F}^{S-T} \otimes \mathbf{X}'^F \right\} = 0, \end{aligned} \quad (72)$$

where the components of tensors Ξ^S, Ξ^F in Cartesian coordinates are given by the relations

$$\Xi_{kKL}^S = \frac{\partial P_{0LL}^S}{\partial F_{kK}^S} F_{IM}^S X_M'^F, \quad \Xi_{kKL}^F = -\frac{\rho^S}{\rho^F} \frac{\partial P_{0LL}^F}{\partial F_{kK}^S} F_{IM}^S X_M'^F. \quad (73)$$

These two conditions contain derivatives of partial stresses with respect to deformations of both components: \mathbf{F}^S in the case of the skeleton and ρ^F in the case of the fluid. Such restrictions of constitutive relations in thermodynamical equilibrium seem to be not plausible. One can eliminate them by assuming

$$\gamma = 0. \quad (74)$$

Then, according to (54), we have

$$\Lambda^{\nu^S} = 0, \quad \Lambda^{\nu^F} = 0. \quad (75)$$

Bearing relations (51), (64), and (70) in mind, we obtain from (74)

$$\eta_d = \frac{1}{T} (\varepsilon_d + \rho_{12}^0) = 0, \quad \text{i.e.} \quad \eta_d = 0, \quad \varepsilon_d = -\rho_{12}^0 = \text{const.} \quad (76)$$

Simultaneously, the relations (65) and (69) imply

$$\sigma^S = -2z\rho_{12}^0, \quad \sigma^F = -2(1-z)\rho_{12}^0. \quad (77)$$

In addition

$$\frac{\partial H_T}{\partial \rho^F} + \Lambda^\varepsilon \frac{\partial K}{\partial \rho^F} = 0, \quad \frac{\partial J_T}{\partial \rho^F} = 0, \quad (78)$$

$$\frac{\partial H_T}{\partial \mathbf{F}^S} + \Lambda^\varepsilon \frac{\partial K}{\partial \mathbf{F}^S} = 0, \quad \frac{\partial J_T}{\partial \mathbf{F}^S} = 0, \quad (79)$$

$$\frac{\partial \Phi}{\partial \rho^F} = 0, \quad \mathbf{F}^{ST} \frac{\partial \Phi}{\partial \mathbf{F}^S} - \Phi \mathbf{1} = 0. \quad (80)$$

The last condition for isotropic materials yields

$$\frac{\partial \Phi}{\partial I} = 0, \quad \frac{\partial \Phi}{\partial II} = 0, \quad \frac{\partial}{\partial J} \left(\frac{\Phi}{J} \right) = 0 \Rightarrow \Phi = \Phi_0 J, \quad (81)$$

where $I, II, III = J^2$ are invariants of \mathbf{C}^S (see: (66)) and Φ_0 is a constant.

iv) The coefficient of $\text{Grad } \mathbf{G} = \text{Grad} \otimes \text{Grad } T$ yields

$$H_T + \Lambda^\varepsilon K = 0, \quad J_T = 0. \quad (82)$$

Relations (78), (79) and (82) give rise to the conclusion that Λ^ε is only a function of T . Then the classical argument on fluxes of energy and entropy leads to the relation

$$\Lambda^\varepsilon = \frac{1}{T}, \quad (83)$$

identifying this multiplier.

v) The coefficient of $\text{Grad } \mathbf{N} = \text{Grad} \otimes \text{Grad } n$ yields

$$H_n - \Lambda^\varepsilon Q_n = 0, \quad J_n = 0. \quad (84)$$

vi) There remains the identity following from the coefficient of $\text{Grad } \Delta_n$. This yields

$$\frac{\partial \rho \eta_0}{\partial \Delta_n} - \Lambda^\varepsilon \frac{\partial \rho \varepsilon_0}{\partial \Delta_n} = 0. \quad (85)$$

We can simplify considerably the above results if we introduce the notion of the Helmholtz free energy defined by the relation

$$\psi = \varepsilon - T\eta, \quad \rho \psi_0 = \rho \psi - \frac{1}{2} \varepsilon_d (\mathbf{F}^S \mathbf{X}'^F) \cdot (\mathbf{F}^S \mathbf{X}'^F), \quad \varepsilon_d = -\rho_{12}^0. \quad (86)$$

Then, bearing the above considerations in mind, we obtain for the Lagrange multipliers the following final results

$$\begin{aligned} \Lambda^{\rho^F} &\equiv \Lambda_0^{\rho^F} = -1/T \frac{\partial \rho \psi_0}{\partial \rho^F}, \quad \Lambda^{\nu^S} = 0, \quad \Lambda^{\nu^F} = 0, \\ \Lambda^\varepsilon &= \frac{1}{T}, \quad \Lambda^n = -1/T \frac{\partial \rho \psi_0}{\partial \Delta_n} = \lambda^n \Delta_n, \quad \Lambda^F = -\frac{1}{T} \frac{\partial \rho \psi_0}{\partial \mathbf{F}^S}. \end{aligned} \quad (87)$$

Obviously, they imply as well

$$\rho \psi_0$$

6. FUNDAM

We are now in
component me
law of thermody
Piola-Kir

$$\mathbf{P}^S =$$

$$\mathbf{P}^F$$

where diffusive

Integration of th

Comparison wit
as a difference
wing these rela
fulfil the classic
contribution to
tion (43).

The Hel

Φ_0, ρ_{12}^0 are cor
Clearly, volume
the fluid and, v
partial stresses i
model.

$$\rho\psi_0 = \rho\psi_E - \frac{1}{2}\lambda^n T\Delta_n^2, \quad \psi_E = \psi_E(\mathcal{R}_E), \quad \lambda^n = \lambda^n(\mathcal{R}_E). \quad (88)$$

6. FUNDAMENTAL RELATIONS OF THE MODEL

We are now in the position to write nonlinear constitutive relations of the two-component medium under considerations which are consistent with the second law of thermodynamics.

Piola-Kirchhoff partial stresses are given by the relations

$$\mathbf{P}^S = \frac{\partial \rho\psi_0}{\partial \mathbf{F}^S} + \left(Q_V - TH_V - \rho^F \frac{\partial \rho\psi_0}{\partial \rho^F} + \lambda^n J\Phi_0 \Delta_n \right) \mathbf{F}^{S-T} - z\rho_{12}^0 \mathbf{F}^S \mathbf{X}'^F \otimes \mathbf{X}'^F, \quad (89)$$

$$\mathbf{P}^F = - \left(Q_V - TH_V - \rho^F \frac{\partial \rho\psi_0}{\partial \rho^F} - \lambda^n J\Phi_0 \Delta_n \right) \mathbf{F}^{S-T} - (1-z)\rho_{12}^0 \mathbf{F}^S \mathbf{X}'^F \otimes \mathbf{X}'^F, \quad (90)$$

where diffusive flux coefficients Q_V and H_V must satisfy the following identity

$$J^2 \frac{\partial}{\partial J} \left(\frac{Q_V - TH_V}{J} \right) = -\rho^F \frac{\partial \rho\psi_0}{\partial \rho^F}, \quad (91)$$

Integration of this relation yields

$$TH_V = Q_V + h(\mathcal{R}_E). \quad (92)$$

Comparison with relations (34) and (36) shows that $h(\mathcal{R}_E)$ can be interpreted as a difference of free enthalpies of both components (see the comment following these relations). Consequently, the energy flux and entropy flux do not fulfil the classical Fourier relation $\mathbf{H} \neq \mathbf{Q}/T$. We do not obtain an anisotropic contribution to the energy flux because we have made the simplifying assumption (43).

The Helmholtz free energy function $\rho\psi_0$ is given by relation (88), Φ_0, ρ_{12}^0 are constant and λ_n, z are arbitrary functions of equilibrium variables. Clearly, volume changes of the skeleton, described by J yield partial stresses in the fluid and, *vice versa*, volume changes of the fluid, described by ρ^F , yield partial stresses in the skeleton. This is the coupling, appearing in the linear Biot model.

Simultaneously, partial stresses in the fluid reduce to the pressure: $\mathbf{P}^F = -p^F \mathbf{F}^{S-T}$, only if we leave out the influence of diffusion described by the coefficient ρ_{12}^0 . Both partial stresses contain a contribution of nonequilibrium changes of porosity Δ_n .

Fluxes of energy, entropy and porosity are given by the relations

$$\begin{aligned} \mathbf{Q} &= Q_\nu \mathbf{X}'^F - K\mathbf{G} + Q_n \mathbf{N}, \\ \mathbf{H} &= H_\nu \mathbf{X}'^F - \frac{K}{T} \mathbf{G} + \frac{Q_n}{T} \mathbf{N}, \\ \mathbf{J} &= \Phi_0 \mathbf{J} \mathbf{X}'^F. \end{aligned} \quad (93)$$

The source of momentum must have the following form

$$\mathbf{F}^{ST} \hat{\mathbf{p}} = \Pi_\nu \mathbf{X}'^F + \Pi_T \mathbf{G} + \Pi_n \mathbf{N} - \rho_{12}^0 \mathbf{F}^{ST} \mathbf{a}_r. \quad (94)$$

These coefficients are limited only by the dissipation inequality. Bearing the above results in mind, we obtain it in the form

$$\begin{aligned} \frac{1}{T^2} K \mathbf{G} \cdot \mathbf{G} - \frac{1}{T} \left(\frac{\partial Q_\nu}{\partial T} - T \frac{\partial H_\nu}{\partial T} - \Pi_T \right) \mathbf{X}'^F \cdot \mathbf{G} + \\ + \frac{1}{T} \Pi_\nu \mathbf{X}'^F \cdot \mathbf{X}'^F - \lambda^n \frac{(\Delta_n)^2}{\tau} \geq 0, \end{aligned} \quad (95)$$

Neither spacial changes of porosity \mathbf{N} nor relative acceleration \mathbf{a}_r contribute to this inequality which means that they yield only reversible effects.

The first term in (95) is, obviously, related to the classical inequality $K \geq 0$ for the heat conductivity coefficient K . The third term yields $\Pi_\nu \geq 0$ which is again the classical result for the diffusion coefficient Π_ν . The remaining coefficients are only weakly limited by this inequality.

7. CONCLUDING REMARKS

Second law of thermodynamics and the assumption on small deviations from thermodynamic equilibrium render the two-component model fairly explicit. The influence of diffusion on the form of free energy function (86) and stress relations (89), (90) are coupled to the parameter ρ_{12}^0 which enters the model through relative accelerations. This is rather supprising. In addition, the term with relative acceleration in the momentum source does not contribute to the dissipation which is also not the result expected by advocates of such models.

The assumption
sion velocity \mathbf{X}
effects (depend
without nonline
ping. An exten
cause the evalu
modynamics is
yields the possi
strated in this v
nonequilibrium

convection. The
contribute to th
upling coefficie
and (90). The c
difference of fr
ilibrium chang
Otherwise both
Helmholtz free

It should
rosity flux \mathbf{J}
Lagrangian rel
changes of the
models in which

The assumption of linearity of the momentum source with respect to the diffusion velocity \mathbf{X}'^F yields only the classical diffusion force without hereditary effects (dependence of permeability on the wave frequency in Biot's model) and without nonlinearities describing such important effects as liquefaction and piping. An extension on such fully nonlinear models would be very difficult because the evaluation of the entropy inequality in contemporary continuum thermodynamics is limited to an infinitesimal vicinity of equilibrium states. This yields the possibility of using Lagrange multipliers technique which we demonstrated in this work. However, even this simple version of the model contains nonequilibrium coupling effects: thermodiffusion Π_V and energy transport by convection. The latter is described by the coefficient Q_V . These coupling effects contribute to the dissipation (94) in the term with the product $\mathbf{X}'^F \cdot \mathbf{G}$. The coupling coefficient plays an important role in the relations for partial stresses (89) and (90). The difference of coefficients $Q_V - TH_V$ which we attributed to the difference of free enthalpies of components is, apart from diffusion and nonequilibrium changes of porosity, the important nonequilibrium part of stresses. Otherwise both partial stresses are described by the potential in the form of Helmholtz free energy function ψ_0 .

It should be also stressed that in spite of nonlinearity of the model the porosity flux \mathbf{J} has a very simple form given by (92)₃. It is proportional to the Lagrangian relative velocity and the coefficient is dependent only on volume changes of the skeleton J . This property has important consequences in linear models in which $J \approx 1$.