

Continuous Modeling of Soil Morphology – Thermomechanical Behaviour of Embankment Dams

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Abstract

Macroscopic modeling of soils is based on a number of properties which refer to the mesoscopic morphology. The most fundamental parameters of this art are

1. coupling parameters between partial stresses of components and deformations of components,
2. porosities
3. saturation
4. permeability and diffusivity, tortuosity.

The main aim of this note is to present in juxtaposition continuous one-, two- and three-component models of geomaterials appearing in construction of embankment dams. In particular the above mentioned features, especially saturation with water and seepage problems, modeling of fluidization yielding piping, generalizations of the Darcy law and changes of porosity are presented.

1 Introduction

Inspection of textbooks and manuals for civil and geotechnical engineers reveals that the design of embankment dams and levees is still based on two issues. It is either a stability analysis based on the one-dimensional Mohr-Coulomb relation

$$\tau = c + \sigma \tan \phi, \quad (1)$$

where τ is the shear strength, σ denotes the normal effective stress on the failure plane and c, ϕ denote the cohesion intercept and the friction angle, respectively, or these are flow nets and streamlines obtained by a graphical, for instance Schmidt's, method (e.g. see Figure 1).

Sometimes it is supplemented by Darcy's law for the estimation of seepage. The rest of those books contains hundreds of examples of existing constructions, a description of their behavior under various loading conditions and failures. Based on this empirical knowledge some heuristic hints for designers are formulated.

This is very different from engineering books in other branches of civil engineering where the design is based on theoretical models which have replaced a sheer collection of observations. This yields as well a very fruitful development of software for computer aided

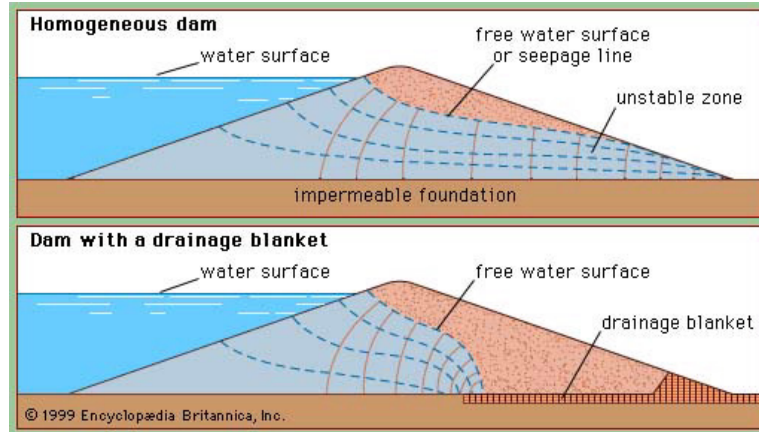


Figure 1: An example of flow nets for two types of embankment dams.

design, such as CAD, Novapoint, etc. M. and I. VANICEK [33] wrote in their book: 'When we look back on the whole process through which the geotechnical engineer has to go, we arrive at the conclusion that the degree of accuracy is significantly lower than for steel or concrete structure, where the differences in the design can be in the order of a few percent, while for earth structures these differences can be in order of tens of percent. That is why on one side the excellent knowledge of soil behavior and treatment of soil as construction material can bring significant savings against conventional design but on the other one disregarding this can lead to failures of earth structures.'

However, the situation is slowly changing to the better because the research in the field of soil mechanics has made a tremendous progress and many theoretical issues such as failure criteria, fluid flow in porous and granular media, heat transfer in soils, micro-macro transitions in theoretical modeling which incorporate porosity changes, saturation, phase changes, dynamics and, particularly, thixotropy or sound wave propagation in soils and rocks were successfully developed.

A choice of theoretical descriptions of aquifers, embankments and many other geotechnical structures depends on the class of phenomena which we want to embrace and on conditions in which the construction or its part should work. For instance, a mechanical loading of a granular dry material yields fragmentation and abrasion. The same mechanical loading of a water saturated granular material yields diffusion, fragmentation but much less abrasion. Hence, in the first case we may expect considerable changes of porosity while in the second case changes of permeability, piping, particle segregation etc. play an important role. This means that the water content in a geomaterial may essentially influence the choice of the theoretical description which is needed.

In this work we present a comparison of fundamental approaches to the theoretical description of the thermomechanical behavior of geotechnical materials. First, we sketch a one-component model with additional internal variables. This may be appropriate for the description of plastic behavior of geomaterials, abrasion but, in many cases of practical interest, also for the description of diffusion. Second, we present a two-component model of a saturated granular material. This model contains a number of additional variables which are able to describe such phenomena as diffusion with variable permeability, lo-

calization of deformation (e.g. on filters) and internal erosion processes. This yields a theoretical description of the backward erosion, concentrated leak and suffusion which are, in turn, main reasons for piping. Finally, also a three-component description for unsaturated granular materials is mentioned.

Thermodynamic modeling of soils developed as a brunch of the theory of immiscible mixtures. This modern continuous approach to systems with multicomponent structures whose one component is solid has been initiated by works of R. M. BOWEN (e.g. [7]). Numerous nonlinear effects in porous and granular materials yield in such a modeling a necessity of application of rather complex mathematical tools. This complexity is then hidden in modern computer programs whose applications in engineering do not require high mathematical skills from software users.

However, such multi-component thermodynamic models of soils require as well an identification of various quantities which do not usually appear in classical continuous models. This concerns, in particular, geometrical properties of such systems, true properties of their components and some process variables characteristic for systems with microstructure. We call this identification the modeling of soil morphology. To name a few examples, one has to identify the porosity, permeability, and for three-component materials the saturation, moisture, capillary pressure and many others. This leads very often to confusion and misinterpretation of results. For this reason, in this work we show fundamental quantities of thermodynamic models of immiscible mixtures appearing in description of soils and their relation to quantities commonly used by soil engineers.

2 One-component modeling of geomaterials

The origin of the one-component models of geomaterials stems from the classical model of elastoplastic materials. They belong to two groups: one describing dry granular materials driven by elastic properties and frictional interactions of grains and the other one describing fully saturated granular materials in which viscosity rather than friction contributes to the mechanical response of the system. Both classes of models contain the macroscopic deformation $\mathbf{B} = B_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$ (the left Cauchy-Green deformation tensor), the velocity $\mathbf{v} = v_i\mathbf{e}_i$ and the temperature θ as unknown functions of the position \mathbf{x} and time t .

In linear models one uses often the displacement vector, $u = u_k\mathbf{e}_k$, related to the Almansi-Hamel deformation tensor, \mathbf{e} , and the velocity, \mathbf{v} , by the following relations

$$\mathbf{e} = e_{kl}\mathbf{e}_k \otimes \mathbf{e}_l, \quad e_{kl} = \frac{1}{2} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right), \quad \mathbf{v} = v_k\mathbf{e}_k, \quad v_k = \frac{\partial u_k}{\partial t}. \quad (2)$$

This deformation measure is related to the left Cauchy-Green tensor by the equation

$$\mathbf{e} = \frac{1}{2} (\mathbf{1} - \mathbf{B}^{-1}). \quad (3)$$

However, these models differ in the set of unknown microstructural variables. The first class contains only the roughness a whose time derivative \dot{a} is called the abrasion, while the second class may contain the abrasion but it must contain also the porosity n and the pore pressure p . For these quantities – fields, we have to construct additional equations.

The classical approach is based on the set of conservation laws of mass, momentum and energy. In Cartesian reference frame they have the following form

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0, \quad (4)$$

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_k} (\rho v_k v_i - \sigma_{ik}) = \rho b_i, \quad (5)$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v_k v_k + \rho \varepsilon \right) + \frac{\partial}{\partial x_k} \left[\left(\frac{1}{2} \rho v_i v_i + \rho \varepsilon \right) v_k + q_k - \sigma_{ki} v_i \right] = \rho b_k v_k, \quad (6)$$

where ρ is the bulk mass density, σ_{ik} are components of the Cauchy stress tensor, $\mathbf{T} = \sigma_{ik} \mathbf{e}_i \otimes \mathbf{e}_k$, b_i are the body forces (e.g. gravitational or centrifugal), $\mathbf{b} = b_k \mathbf{e}_k$, ε is the specific internal energy and q_k are components of the heat flux vector, $\mathbf{q} = q_k \mathbf{e}_k$. It is often assumed that the real grains of the material are incompressible. This means that the true mass density ρ^{SR} , $\rho = (1 - n) \rho^{SR}$ (S for 'solid' and R for 'real' or 'true'; in soil mechanics one denotes sometimes $\rho^{SR} = \gamma$), is constant. The porosity n is related to the void ratio e , $0 \leq e < \infty$, often used in soil mechanics, by the simple relation $n = e/(1 + e)$, $0 \leq n \leq 1$.

All these arguments are related to the two-component model which we discuss in Section 3. Therefore, the changes of porosity following from the incompressibility assumption cannot be consistently incorporated in a one-component model. The following relation, used in this model,

$$n = n_0 \left(1 + \frac{1 - n_0}{n_0} e \right) \quad (7)$$

where n is the current porosity, n_0 the initial porosity and $e = \text{tr}(\mathbf{1} - \mathbf{B}^{-1})$ the volume changes of the solid frame (caution: this e is not the same as the void ratio, mentioned above), must be considered as an additional assumption of the one-component model.

We return to the microscopic interpretation of these quantities in Section 4.1 of the present work.

In order to obtain the equations of the model of dry granular materials we have to specify constitutive relations for stress tensor σ_{ik} , internal energy ε , heat flux q_k and the abrasion \dot{a} .

Experience shows that granular materials behave plastically. Consequently, the classical Mohr-Coulomb relation has been extended to relate the stress tensor and the deformation tensor. As the so-called hardening effects play an important role in such models one had to introduce additional internal variables (the so-called back-stress). The result is the cam-clay model commonly used in the literature on soil mechanics (e.g. see: D. MUIR WOOD [45], [46], LANCELLOTTA [27]). As an alternative a so-called hypoplasticity was introduced (BAUER [2], WOLFFERSDORF [44], KOLYMBAS [24], [25]). In contrast to the cam-clay model the hypoplasticity is rate-dependent which means that the rate of deformation $D_{ij} = \frac{1}{2} [\partial v_i / \partial x_j + \partial v_j / \partial x_i]$ (it is related to the time derivative of the deformation tensor B_{ij}) has an influence on the current values of the stress. The general constitutive relation for stresses in this model has the form

$$\dot{\sigma}_{ij} = f_{ij} [\sigma_{ij}, D_{ij}, n], \quad (8)$$

where $\hat{\sigma}_{ij}$ denotes an objective time derivative of the stress tensor.

We do not need to go into details of these models in the present work. Many of them can be found in BAUER [3].

It remains to specify the internal energy ε , the heat flux q_k and the abrasion \dot{a} . For the first two quantities one usually assumes that the classical Fourier model of heat conduction is valid. This may be questionable in some fast processes but for the thermomechanical description of embankments it seems to be sufficient. For the purpose of our analysis it is sufficient to point out the ways in which energy is transported in the medium. They are specified in (6) by contributions under the div operator (i.e. $\frac{\partial}{\partial x_k}$). The first one is the convection. The second one, described by \mathbf{q} , consists of two parts: the conduction, \mathbf{q}_c and the diffusion, \mathbf{q}_d with $\mathbf{q} = \mathbf{q}_c + \mathbf{q}_d$. The latter means that the energy is transported by the relative motion of components. Such a mechanism cannot be described by a one-component model. The conduction is related to the transfer of energy due to the temperature gradient. Finally, the last contribution, $\mathbf{T}\mathbf{v}$, is the bulk working of stresses which is also of no interest in this work.

Conduction in isotropic materials is usually described by the linear Fourier law

$$\mathbf{q}_c = -\lambda \text{grad}\theta, \quad (9)$$

where θ is the absolute temperature. The coefficient λ , the heat conductivity, is for soils heavily dependent on the morphology. This dependence cannot be reflected by a one-component model either. We return to this problem in Section 4.3.

The form of the equation for abrasion has a long history and it goes back to the work of GOODMAN and COWIN [19]. However, in this pioneering work the equation was proposed rather for changes of volume fraction than for the abrasion. It was first the series of works of K. HUTTER (e.g. [34], [35]) and the PhD Thesis of N. KIRCHNER [23] where this equation was thermodynamically justified. Its form follows from the assumption that microstructural changes of the configuration caused by the abrasion must be accompanied by the so-called configurational forces. Then the abrasion \dot{a} , corresponding to the classical notion of momentum, must satisfy a balance law which is assumed to have the form

$$\rho k \frac{\partial^2 a}{\partial t^2} = \frac{\partial h_i}{\partial x_i} + \rho(l + f), \quad (10)$$

where k is the material parameter describing the resistance of the material to changes of its internal surface. According to KIRCHNER [23]: 'change of surface properties includes the smoothening of initially rough grain surface (that is, $\dot{a} < 0$) as well as the roughening of initially smooth grain surfaces (that is, $\dot{a} > 0$)'. h_i is the surface stress of abrasion. l and f are supply and production, respectively and the latter must be given by a constitutive law of its own.

The above sketched one-component model of dry geomaterials is often extended by an equation describing the flow of water through the saturated granular material. All equations described above are assumed to remain unchanged. The seepage process through the saturated material is supposed to satisfy some additional balance law which is justified experimentally. Such a justification goes back on works of DARCY [12] and it has been incorporated in soil mechanics by VON TERZAGHI [32]. In the local form this law can be written in the form

$$Q_i = -\frac{k_{ij}}{\mu} \frac{\partial p}{\partial x_j}, \quad (11)$$

where Q_i is the so-called specific discharge, i.e. a relative velocity of water with respect to the skeleton, k_{ij} is the matrix of permeability which reduces to a scalar k for isotropic materials and μ is the kinematic viscosity of water. It has been shown that such a relation holds for small relative velocities which are connected with a laminar flow of the water (low Reynolds number, $Re < 1 \div 10$, J. BEAR [4]). In the case of fast flows yielding turbulence (high Reynolds numbers) the Darcy law does not hold. Most likely FORCHHEIMER [16] was the first who proposed nonlinear corrections to (11) in order to describe such flows. They play a particularly important role in processes of piping, commonly appearing in embankment dams. A more rational procedure of description of seepage is proposed by theories of multicomponent systems.

3 Two-component modeling of geomaterials

The thermomechanical model of a one-component geomaterial can be considerably improved when one applies a theory of immiscible mixtures. We shall do so first for fully saturated materials. In this case the extension yields a better physical insight but for purposes of geotechnics it is not necessary. However, for partially saturated materials such extensions are unavoidable and, simultaneously, they are similar to two-component models in many technical details. We will see this in the following sections on multi-component modeling.

In the case of two components one has to describe the partial macroscopic fields for each component. These are partial mass densities ρ^S, ρ^F with the bulk mass density $\rho = \rho^S + \rho^F$, partial velocities $\mathbf{v}^S, \mathbf{v}^F$ with the bulk (barycentric) velocity $\mathbf{v} = (\rho^S/\rho) \mathbf{v}^S + (\rho^F/\rho) \mathbf{v}^F$, partial Cauchy stresses $\mathbf{T}^S = \sigma_{ij}^S \mathbf{e}_i \otimes \mathbf{e}_j$, $\mathbf{T}^F = \sigma_{ij}^F \mathbf{e}_i \otimes \mathbf{e}_j$ with the bulk stress $\mathbf{T} \approx \mathbf{T}^S + \mathbf{T}^F$. These quantities must satisfy balance laws

$$\frac{\partial \rho^\alpha}{\partial t} + \frac{\partial}{\partial x} (\rho^\alpha v_i^\alpha) = 0, \quad \alpha = S, F, \quad (12)$$

$$\frac{\partial (\rho^\alpha v_i^\alpha)}{\partial t} + \frac{\partial}{\partial x_k} (\rho^\alpha v_i^\alpha v_k^\alpha) = \frac{\partial \sigma_{ik}^\alpha}{\partial x_k} + \hat{p}_i^\alpha + \rho^\alpha b_i^\alpha, \quad \hat{p}_i^S + \hat{p}_i^F = 0. \quad (13)$$

It is easy to check that conservation laws of a one component model are then identically satisfied provided we neglect quadratic terms in relative velocities $\mathbf{v}^\alpha - \mathbf{v}$ which seems to be well justified for processes in soils far from the structural loss of stability such as fluidization. The momentum source $\hat{\mathbf{p}}^S = \hat{p}_i^S \mathbf{e}_i = -\hat{\mathbf{p}}^F = -\hat{p}_i^F \mathbf{e}_i$ is related to the diffusion force. In the case of an isotropic model linear in relative velocities it can be written in the form

$$\hat{\mathbf{p}}^S = \pi (\mathbf{v}^F - \mathbf{v}^S), \quad (14)$$

where π is the permeability coefficient. In the case of water one can assume that the partial stress tensor \mathbf{T}^F is spherical, i.e. it reduces to the partial pressure p^F . Then the momentum balance for the fluid written in Cartesian coordinates has the form

$$\rho^F \left(\frac{\partial v_i^F}{\partial t} + v_k^F \frac{\partial v_i^F}{\partial x_k} \right) = -\frac{\partial p^F}{\partial x_i} + \pi (v_i^F - v_i^S) + \rho^F b_i^F. \quad (15)$$

This equation yields Darcy's law for processes with small changes of porosity and negligible

inertial forces. In such a case, it follows

$$p^F = n_0 p, \quad (v_i^F - v_i^S) = -\frac{n_0}{\pi} \frac{\partial p}{\partial x_i}, \quad (16)$$

where n_0 is the initial porosity and $n \approx n_0$. This is identical with (11) for isotropic materials with an appropriate definition of the permeability $k/\mu = n_0/\pi$. Hence the two-component model yields the one-component model as a particular case.

On the other hand, the general form of partial momentum balance (12) admits also nonlinear contributions which yield the loss of stability of the fluid motion. If changes of porosity and relative velocity are not small one can introduce the following relation for the source of momentum (see: WILHELM, WILMANSKI [37])

$$\hat{p}_i^S = \pi (v_i^F - v_i^S) - \left(p + \rho^S \frac{\partial \psi^S}{\partial n} \right) \frac{\partial n}{\partial x_i}, \quad (17)$$

where p is the pore pressure and ψ^S is the Helmholtz free energy of the solid component dependent on deformations, porosity and relative velocity. This form of the source is justified by thermodynamic considerations which we shall not discuss in this work. The simplest choice of the dependence on the relative velocity which yields piping is as follows

$$\begin{aligned} \rho^S \frac{\partial \psi^S}{\partial n} &= \frac{\Gamma}{\sqrt{2}} \left(1 + \frac{W - Y}{|W - Y|} \right) \sqrt{W}, \\ \Gamma, Y &> 0, \quad W = \frac{1}{2} (v_i^F - v_i^S) (v_i^F - v_i^S), \end{aligned} \quad (18)$$

where Γ is a material parameter and Y is the threshold velocity. As shown in [37] this model yields a quantitative agreement with experiments on sands.

3.1 Changes of porosity

Changes of porosity may be described by the relation (7) following from the assumption on incompressibility of grains. However, there is an evidence stemming from poroacoustics that such an assumption eliminates an important $P2$ -wave from the model (e.g. WILMANSKI [42]). Many other problems must also account for changes of porosity. For instance, within soil mechanics these are large plastic deformations (e.g. E. BAUER [2]), damage (for instance, in freezing and thawing) or, for granular materials, combustion problems of solid fuels.

Generally, there are a few different approaches to changes of porosity. We mention here the five most commonly appearing in the literature:

1. Constitutive assumption, e.g. equilibrium changes coupled on volume changes of skeleton (F. GASSMANN [17]),
2. Incompressibility assumption (R. M. BOWEN [7]),
3. Evolution equation (R. M. BOWEN [8]),

4. Second order equation based on a principle of equilibrated pressure (M. GOODMAN AND S. COWIN [19], extended by PASSMAN, NUNZIATO AND WALSH [29], K. HUTTER et al. [14]),
5. Balance equation (K. WILMANSKI [38], [39]).

Gassmann's model follows from a simplified micro-macro description and results in the relation

$$n = n_0 (1 + \delta e), \quad (13)$$

where e denotes small volume changes of the skeleton and δ is a material parameter related to compressibilities of components. n_0 is the initial porosity. We show some properties of this model further in this Section.

The assumption of incompressibility which is essential for Bowen's approach [7] states

$$\rho^{SR} = \text{const.}, \quad (14)$$

which yields the following form of the macroscopic mass balance equation for the skeleton

$$\rho^{SR} \frac{\partial}{\partial t} (1 - n) + \rho^{SR} \text{div} [(1 - n) \mathbf{v}^S] = 0, \quad (15)$$

provided there is no mass exchange with other components.

Easy integration of this equation yields for small deformations

$$n = n_0 \left(1 + \frac{1 - n_0}{n_0} e \right), \quad (16)$$

which is, obviously, equation (7) of the one-component model and it reminds Gassmann's relation (13) but there is no relation to material parameters. In this case $\delta = (1 - n_0)/n_0$. We will not discuss the next two models and mention only that the evolution equation proposed by Bowen is a particular case of the balance equation of porosity, at least for small deformations, when one neglects the influence of diffusion. The Goodman and Cowin proposition is related to some microstructural considerations which have a bearing in the case of combustion problems for powders. K. Hutter and his coworkers have shown that some extensions of this model describe well the behavior of snow avalanches and mud flows.

The porosity balance of the last model, mentioned above, is thermodynamically admissible and yields a consistent model for large deformations (K. WILMANSKI [43]). As we will see below, in the case of small deformations of soils this equation can be immediately solved. Its linear version for two-component systems has the form

$$\begin{aligned} \frac{\partial \Delta_n}{\partial t} + \Phi \text{div} (\mathbf{v}^F - \mathbf{v}^S) &= -\frac{\Delta_n}{\tau}, \\ \Delta_n = n - n_E, \quad n_E &= n_0 (1 + \delta e), \end{aligned} \quad (17)$$

where \mathbf{v}^F is the macroscopic velocity of the fluid, τ, δ, Φ are material constants. In the case of soils, one can usually neglect relaxation effects described by the right-hand side of

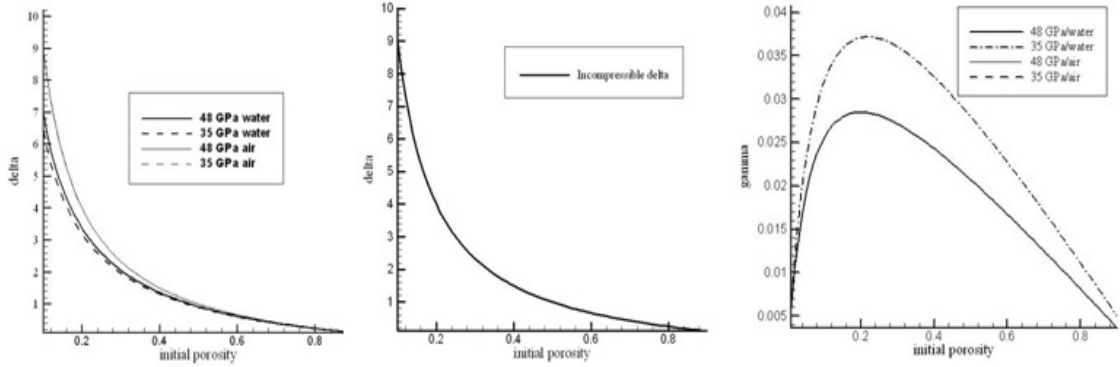


Figure 2: Coefficients δ and γ of the linear porosity relation without source. Left: coefficient δ for compressibilities of the skeleton 35 and 48 GPa with water or air filling the pores, middle: δ for incompressible skeleton ($\rho^{SR} = \text{const.}$), right: coefficient γ for nonequilibrium contribution.

the equation. This means that one can take the limit $\tau \rightarrow \infty$. Then, the equation can be integrated and the following relation follows

$$n = n_0 (1 + \delta e + \gamma (e - \varepsilon)), \quad \gamma = \frac{\Phi}{n_0}, \quad (18)$$

where ε is the volume change of the fluid component. The quantity $\zeta = (e - \varepsilon)/n_0$ is called the increment of fluid content. The above relation can be easily extended to three components (see: B. ALBERS [1]). We shall not present this equation in this work.

Similarly to the Gassmann relation, it can be shown that material parameters δ, γ can be identified by means of compressibilities of components (K. WILMANSKI [40]). Simultaneously, they coincide with coefficients of porosity changes predicted by Biot's model (M. A. BIOT [6]). In Figure 2, we present a few examples of the behavior of these parameters. The left panel shows the dependence of the coefficient δ (equilibrium changes of porosity) as a function of the initial porosity, n_0 . The curves correspond to the compressibility of the solid skeleton 48 GPa and 35 GPa, respectively. The two lower curves are plotted for water and the two coinciding upper curves are plotted for air. The middle panel shows the illustration of δ for an incompressible skeleton (relation (16); in this case $\gamma \equiv 0$). There is almost no difference between this curve and the curves for air in the left panel. Finally, the right panel shows the behavior of γ which reflects the influence of diffusion on changes of porosity. Obviously, this influence is rather small and the values for the saturation with air are so small that the curves are not visible in the scale of Figure 2.

3.2 Tortuosity

The notion of tortuosity contributes to numerous confusions in modeling porous media. It is still disputable in what way one should include this measure of complexity of microstructure.

The tortuosity, τ , is taken as the ratio of the average pore length to the macroscopic characteristic length along, for instance, the major flow (i.e. the longest local streamline).

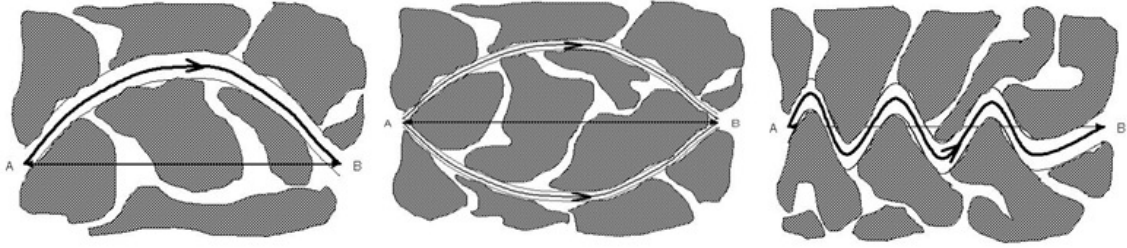


Figure 3: Different tortuosities for three morphologies of the same porosity.

An example of different values of tortuosity for a medium with the same local porosity is exemplified in Figure 3. The left situation yields the smallest value of τ and the right situation – the largest value of τ among these three cases. The value of τ is the ratio of the distance of points A and B to the length of channels indicated by the arrows.

As shown by N. EPSTEIN [13] the tortuosity influences the intensity of diffusion by entering a material parameter relating the pressure gradient and the diffusion velocity. We return to this coefficient in the next Section. However, it should be mentioned that it is the square of τ which appears in this relation. The error of the linear dependence made by J. KOZENY [26] has been corrected in many works and the quadratic dependence seems to be well established.

In many papers on acoustics of porous media it is claimed that the tortuosity enters the model by the so-called added mass effect. This has been introduced by M. A. BIOT [5] in the form of a off-diagonal contribution ρ_{12} to the partial mass matrix. It can be understood as a coupling of components through inertial forces. One can show in a simple thermodynamic analysis (K. WILMANSKI [41]) that such a coupling is nondissipative. This means that the tortuosity cannot have an influence on the damping of acoustic waves. This is, of course, a nonsensical conclusion. This contribution, can be indeed introduced to poroelastic models after some nonlinear corrections, but it cannot be interpreted as an influence of tortuosity.

3.3 Permeability

The notion of permeability of porous media is usually related with the Darcy law which expresses a total discharge of fluid in a one-dimensional flow in terms of the pressure difference. However, there is still a bit of confusion in the terminology. We use in the model of porous materials the term of a coefficient of permeability or coefficients of permeability if we deal with more than one fluid component. We proceed to introduce these notions in a systematic way.

The fundamental notion of intrinsic permeability, κ , is related solely to the morphology of a porous skeleton and it is independent of the kind of fluid in the channels. It may be related to the effective diameter of the pores, d , by the relation

$$\kappa = C d^2, \quad (19)$$

where C is a dimensionless constant. The units of κ are $[\text{m}^2]$ but in practical applications often the unit $1 [\text{darcy}] \equiv 1 \text{ D} \approx 10^{-12} [\text{m}^2]$ is used.

In the second column of Table 1 we quote some typical values of this parameter.

Table 1: Intrinsic permeability, κ and hydraulic conductivity, K for some formations

soil	κ [darcy]	K [m/s]
well sorted gravel	$10^5 - 10^2$	$1 - 10^{-3}$
oil reservoir	$10 - 10^{-1}$	$10^{-4} - 10^{-6}$
sandstone	$10^{-2} - 10^{-3}$	$10^{-7} - 10^{-8}$
granite	$10^{-6} - 10^{-7}$	$10^{-11} - 10^{-12}$

In order to characterize the flow of a particular fluid one has to account also for properties of this fluid. If the kinematic viscosity is μ and its mass density ρ^{FR} then the parameter

$$K = \frac{\kappa\gamma^{FR}}{\mu}, \quad \gamma^{FR} = \rho^{FR}g, \quad (20)$$

is called the hydraulic conductivity; g is the earth acceleration. A few typical values for water (pressure 10^5 Pa, temperature 20^0 C, $\mu = 1.002 \times 10^{-3}$ Pa·s) are quoted also in Table 1. Then the typical form of the Darcy law is as follows

$$Q(\rho^{FR}g) = KA\frac{\Delta p}{L}, \quad (21)$$

where Q [m^3/s] is the total discharge through the surface A , $\Delta p = p_a - p_b$ is the pressure difference and L is the distance between two faces a and b .

In addition, for horizontal aquifers one uses the notion of transmissivity, T , which is the product of the hydraulic conductivity, K , and the thickness, d , of the aquifer.

If we refer the diffusive flow to the difference of the concentration rather than to the difference of the pressure – one speaks then about the first Fick law – then the diffusion flux and the gradient of concentration are connected by the diffusion coefficient, D . This notion is, of course, related to the hydraulic conductivity, K , by a simple change of variables.

Finally, let us remark that the constant C of the relation (19) is dependent on the porosity and on the tortuosity (a quadratic dependence as we argued before).

Theories of porous media are based on the model of immiscible mixtures. Then Darcy's law does not enter the model at all. It is replaced by partial momentum equations. In the case of a linear model of two components the simplest form of this equation is the following

$$\rho^F \frac{\partial \mathbf{v}^F}{\partial t} = -\text{grad}p^F + \pi(\mathbf{v}^F - \mathbf{v}^S). \quad (22)$$

If we neglect the acceleration it becomes a precursor of Darcy's law. The coefficient π which is inversely proportional to the hydraulic conductivity is called the coefficient of permeability.

The microstructural justification of the above relations for parameters of permeability is difficult. Such laws are known from the kinetic theory of mixture of gases and, in the case of granular materials, they are mimicked by assuming that grains have a very big mass in comparison to the gas in pores and, consequently, can be assumed to be immobile. One

obtains the so-called gas-dust diffusivity (e.g. J. A. WESSELINGH, R. KRISHNA [36]) which is the counterpart of the hydraulic conductivity. Some results, also experimental, are obtained for regular geometries of channels. Otherwise, one has to rely on purely macroscopic observations.

We leave out considerations concerning heat conduction. This problem shall be presented in some details in Section 3 on three-component models.

One can conclude the above remarks that the two-component model may play an important role in geotechnics for processes in which nonlinearities are essential. This concerns large deformations and, consequently, large changes of porosity and permeability. Of particular importance are, however, large relative (seepage) velocities which yield the loss of stability and piping. Otherwise one-component models seem to be acceptable for both dry and wet granular materials of geotechnical bearing.

4 Three-component modeling of geomaterials

As indicated in the previous Section, for three-component continuous models besides the porosity at least one more microstructural variable appears, namely the saturation, i.e. the fraction of the volumes of one of the pore fluids to this of the void space. Then, in the thermodynamical analysis the following fields have to be considered:

1. partial mass densities, ρ^S, ρ^F, ρ^G ,
2. velocities of components, $\mathbf{v}^S, \mathbf{v}^F, \mathbf{v}^G$,
3. common temperature of components, T ,
4. porosity, n ,
5. saturation, S .

The physical significance of the first and second group will be explained in the following subsection.

4.1 Micro-macro transitions for porous materials

The construction of thermodynamic models of materials with small randomly distributed voids requires always a smearing-out procedure which transforms functions on complicated and different domains to a common domain, say \mathcal{B}_0 , which, in turn, is identified with a continuous porous medium. Such constructions for multicomponent systems with a relative motion (diffusion) yield the so-called immiscible mixtures. Quantities appearing in such models are not directly measurable and require, as we indicated in the introduction to this work, certain identification rules. In some cases these rules can be simply constructed by volume averaging and in some other cases they require more sophisticated methods of identification. We begin our presentation with a few typical quantities which follow from the former procedure.

The volume averaging is performed by means of the so-called Representative Elementary Volume (*REV*, e.g. J. BEAR [4]) consisting of points \mathbf{Z} . *REV* is small enough to be replaced by a material point \mathbf{X} of \mathcal{B}_0 at which a set of macroscopic quantities replaces real (true) quantities of the microdomain *REV*. For three-component systems which we consider in this section, the following quantities are defined by averaging over *REV* (\mathbf{X}, t) prescribed to the point $\mathbf{X} \in \mathcal{B}_0$ at the instant of time t :

1. Porosity

$$n(\mathbf{X}, t) = 1 - \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{X}, t)} \chi^S(\mathbf{Z}, t) dV_Z,$$

$$V_S = \int_{REV(\mathbf{X}, t)} \chi^S(\mathbf{Z}, t) dV_Z, \quad (23)$$

where $\chi^S(\cdot, t)$ is the characteristic function of the skeleton, i.e. it is equal to one if the point \mathbf{Z} is occupied by a particle of the skeleton (solid phase) at the instant of time t , and zero otherwise, and $\text{vol}(REV) = V$ is the volume of REV . In continuous models, the porosity is usually identical with the porosity available for the transport in pores. This means that, for instance, contributions of dead-end channels are not included in n . The latter contribute to changes of the true (real) effective properties of the skeleton.

2. Mass density of the skeleton

$$\rho^S(\mathbf{X}, t) = \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{X}, t)} \rho^{SR}(\mathbf{Z}, t) \chi^S(\mathbf{Z}, t) dV_Z, \quad (24)$$

where $\rho^{SR}(\mathbf{Z}, t)$ is the real (true) mass density of the skeleton at the point \mathbf{Z} and at the instant of time t .

3. Mass density of the fluid

$$\rho^F(\mathbf{X}, t) = \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{X}, t)} \rho^{FR}(\mathbf{Z}, t) \chi^F(\mathbf{Z}, t) dV_Z,$$

$$V_F = \int_{REV(\mathbf{X}, t)} \chi^F(\mathbf{Z}, t) dV_Z, \quad (25)$$

where $\rho^{FR}(\mathbf{Z}, t)$ is the real (true) mass density of the fluid at the point \mathbf{Z} and at the instant of time t , and $\chi^F(\cdot, t)$ is the characteristic function of the fluid, i.e. it is equal to one if the point \mathbf{Z} is occupied by a particle of the fluid at the instant of time t , and zero otherwise.

4. Mass density of the gas

$$\rho^G(\mathbf{X}, t) = \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{X}, t)} \rho^{GR}(\mathbf{Z}, t) \chi^G(\mathbf{Z}, t) dV_Z,$$

$$V_G = \int_{REV(\mathbf{X}, t)} \chi^G(\mathbf{Z}, t) dV_Z, \quad (26)$$

where $\rho^{GR}(\mathbf{Z}, t)$ is the real (true) mass density of the gas at the point \mathbf{Z} and at the instant of time t , and $\chi^G(\cdot, t)$ is the characteristic function of the gas, i.e. it is equal to one if the point \mathbf{Z} is occupied by a particle of the gas at the instant of time t , and zero otherwise.

In many cases of practical importance, one assumes a microhomogeneity of the microstructure which means that true mass densities are approximately constant on REV . Then

$$\begin{aligned}\rho^S &= (1 - n)\rho^{SR}, & \rho^F &= nS\rho^{FR}, \\ \rho^G &= n(1 - S)\rho^{GR}, & n &= \frac{V_F + V_G}{V},\end{aligned}\tag{27}$$

where S denotes the saturation

$$S = \frac{V_F}{V_F + V_G},\tag{28}$$

and $\rho^{SR}, \rho^{FR}, \rho^{GR}$ are evaluated in an arbitrary point $\mathbf{Z}_0 \in REV(\mathbf{X}, t)$ (microhomogeneity!).

Sometimes the product

$$\theta = nS \equiv \frac{V_F}{V}, \quad V = V_S + V_F + V_G,\tag{29}$$

is called the volumetric water content. This should be distinguished from the gravimetric water content (a moisture fraction)

$$\begin{aligned}w &\equiv m_W = \frac{M_F}{M_S} = \frac{\rho^{FR}V_F}{\rho^{SR}V_S}, \\ V_L &= nSV, \quad V_S = (1 - n)V,\end{aligned}\tag{30}$$

i.e. it is the fraction of the mass of fluid to the mass of the (real, true, dry) skeleton. Certainly, bearing relations (27) in mind, it can be written in the form

$$m_W = \frac{\rho^F}{\rho^S}.\tag{31}$$

Consequently, in contrast to porosity, n , and saturation, S , neither the volumetric water content, θ , nor the moisture fraction, m_w , are independent microstructural quantities. However, the latter is sometimes used as a measure of compactness of soils reflected by the partial mass density of the skeleton, ρ^S .

The construction of volume averages illustrated above fails in the case of transport coefficients. This concerns both classical coefficients such as heat conductivity or hydraulic conductivity, as well as partial stresses. In the latter case, one makes sometimes simplifying assumptions

$$\begin{aligned}p^S &= (1 - n)p^{SR}, & p^F &= nSp^{FR}, \\ p^G &= n(1 - S)p^{GR}, & p^L &= p^F + p^G = np^{LR}, \\ p^{LR} &= Sp^{FR} + (1 - S)p^{GR},\end{aligned}\tag{32}$$

where $p^S = -1/3\text{tr}\mathbf{T}^S$ is the partial pressure in the skeleton (\mathbf{T}^S denotes the partial Cauchy stress tensor in the skeleton) and p^{SR} denotes the true (real) pressure in the skeleton, p^F and p^{FR} are partial and, respectively, real pressure in the fluid, p^G and p^{GR} – in the gas, and p^{LR} is the pore pressure. One can show that relations (32) are a combination of volume averaging under very restrictive conditions and Dalton's law for

fluid components. However, one should bear in mind that they may not hold, for instance, near boundaries or for dynamical processes such as shock waves. The following quantity

$$p_c = p^{GR} - p^{FR}, \quad (33)$$

is identified with the capillary pressure and used in constitutive relations for the saturation, S .

Fields of velocities of components are more difficult to interpret in terms of microstructural quantities. It is obvious that deviations of real velocities of the skeleton and of the fluid components from their macroscopic counterparts, both with respect to directions and the magnitude, are usually very large. Such fluctuations cannot be easily estimated by averaging. Hence, the terminology: diffusion velocities $\mathbf{v}^F - \mathbf{v}^S$, $\mathbf{v}^G - \mathbf{v}^S$, with respect to the skeleton – or frequently appearing in soil mechanics – seepage velocity or velocity of filtration should be understood as macroscopic notions and their interpretation may change from one process to the other. For instance, in cases of catastrophic phenomena such as liquefaction the filter velocity cannot be identified with any of the macroscopic average velocities. Macroscopic models based on the above notions must be then correspondingly extended.

At the first glance, the problem of averaging of velocities can be replaced by the averaging of momenta. These are volume densities and, consequently, one could write

$$\begin{aligned} \rho^S \mathbf{v}^S &= \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{x},t)} \rho^{SR}(\mathbf{Z},t) \mathbf{v}^{SR}(\mathbf{Z},t) \chi^S(\mathbf{Z},t) dV_{\mathbf{Z}}, \\ \rho^F \mathbf{v}^F &= \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{x},t)} \rho^{FR}(\mathbf{Z},t) \mathbf{v}^{FR}(\mathbf{Z},t) \chi^F(\mathbf{Z},t) dV_{\mathbf{Z}}, \\ \rho^G \mathbf{v}^G &= \frac{1}{\text{vol}(REV)} \int_{REV(\mathbf{x},t)} \rho^{GR}(\mathbf{Z},t) \mathbf{v}^{GR}(\mathbf{Z},t) \chi^G(\mathbf{Z},t) dV_{\mathbf{Z}}, \end{aligned} \quad (34)$$

where \mathbf{v}^{SR} , \mathbf{v}^{FR} , \mathbf{v}^{GR} denote real (true) velocities in channels. However, this is not much of the help. First of all, the macroscopic momentum balance equations which yield field equations for \mathbf{v}^S , \mathbf{v}^F , \mathbf{v}^G must contain source terms such as diffusive forces reflecting interactions through walls of channels or frictional forces between fluid components. Such terms are not present in microstructural momentum balance equations and must be introduced by some additional surface or line integrals on REV which, of course, has nothing to do with volume averaging. Secondly, average momentum densities do not describe strong deviations (fluctuations) of true momenta from average values anyway. These would have to be introduced additionally to averaged momentum balance equations. One of the microstructural variables designed for this purpose is the tortuosity described in Section 3.2. Consequently, in spite of a few attempts to develop this procedure it seems to be easier to deal directly with purely macroscopic models.

The assumption on a common temperature of components, T , is related with difficulties of thermodynamic procedures for mixtures with multiple temperatures. Such procedures are scarce in literature and usually not directly applicable to porous media (e.g. see: T. RUGGERI, S. SIMIĆ [30], D. IEŞAN [22]).

The field of porosity, n , has been discussed in detail in the last Section about two-component models. In the following subsection, the notion of saturation, S , is discussed in some details.

4.2 Capillary pressure, saturation, retention curves

As mentioned earlier, the construction of a relation for the saturation as one of the fields for a three-component immiscible mixture is based on considerations concerning mechanical properties of the microstructure. They are still limited to isothermal conditions.

Between two immiscible fluids, of which one may be gaseous, e.g. water and air, a discontinuity in pressure exists across the interface separating them. The difference is called capillary pressure (compare (33))

$$p_c = p^{GR} - p^{FR}, \quad (35)$$

where p^{GR} is the true pressure in the nonwetting phase (gas in our case) and p^{FR} the corresponding value in the wetting fluid (water in our case). The fluid phase whose molecules or atoms preferentially are adsorbed on a solid surface is called the wetting fluid while the superseded material is denoted as nonwetting fluid.

In a fluid-gas-mixture, the Young-Laplace equation describes the capillary pressure difference due to the phenomenon of surface tension σ and relates it to the radius of the bubbles r

$$p_c = 2\frac{\sigma}{r}. \quad (36)$$

For a porous medium, the capillary pressure is a measure of the tendency to suck in the wetting fluid or to repel the nonwetting phase. In soil science, the negative of the capillary pressure (expressed as the pressure head) is called suction. The radius r then is of the order of magnitude of the pore or grain size. The capillary pressure, thus, depends on the geometry of the void space, on the nature of the solids and fluids and on the degree of saturation. As we have already pointed out, in soils, the geometry of the void space is extremely irregular and complex. Hence, only an idealized model may be adopted (e.g. capillary tubes, spheres of constant radius or a bundle of parallel circular rods). Laboratory experiments are probably the only method to derive the relationship $p_c = p_c(S)$.

In the experimentally determined capillary pressure curves a hysteresis occurs. This means that different capillary pressures may be obtained for a certain degree of saturation, depending on whether a sample is initially saturated with a wetting or with a nonwetting fluid. In both cases the fluid initially saturating the sample is slowly displaced by the other fluid. When the sample is initially saturated with a wetting fluid, the process is called drainage, otherwise imbibition. Figure 4 which is taken from J. BEAR [4] shows a typical capillary pressure - wetting fluid saturation relationship (kerosene and water in a sandstone) including the effect of hysteresis. In theoretical approaches the hysteresis is neglected for simplification. The capillary pressure curve is known also under several other denotations, e.g. it is also called retention curve, pF-curve or soil-water characteristic curve.

Inspection of Figure 4 shows that a certain quantity of wetting fluid remains in the sample even at high capillary pressures. The value of the water saturation at this point (Bear

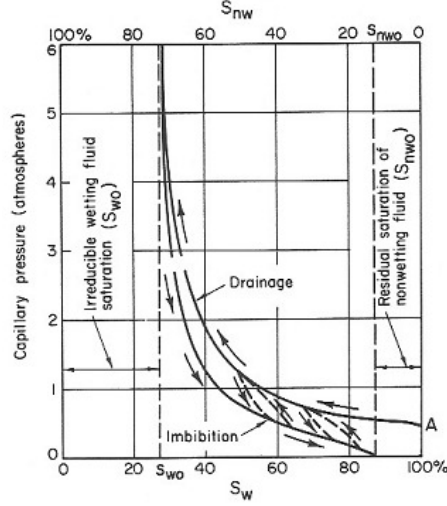


Figure 4: Typical capillary pressure-wetting fluid saturation curves illustrating hysteresis. (J. BEAR).

denotes it by S_{w0} otherwise it is often described by $S^{rF} - r$ stands for residual) is called irreducible saturation of the wetting fluid. Something similar appears if one looks at the imbibition curve. It is observed that at zero capillary pressure there remains a certain amount of the nonwetting fluid – the residual saturation of the nonwetting fluid (Bear denotes it by S_{nw0} otherwise it is often described by S^{rG}). It indicates the amount of entrapped air in the pores. This leads to the introduction of the term effective saturation (for more information see e.g. R. HELMIG [21])

$$S_e := \frac{S - S^{rF}}{1 - S^{rF} - S^{rG}} \quad \text{or} \quad S_e := \frac{S - S^{rF}}{1 - S^{rF}}. \quad (37)$$

The measured capillary pressure curves for several soil types differ considerably. Particularly, the range of the saturation which in a special soil can appear is rather distinct. A rough overview for sands, silts and clays is given in Figure 5 which is a modification of a figure of K. H. HARTGE & R. HORN [20]. The entire curve, according to P. SCHICK [31], can only be obtained by combining several measuring techniques because the range of capillary pressures can cover up to seven orders of magnitude while each technique is applicable only up to three orders of magnitude.

There are some attempts to put the once experimentally measured curves into formulae. The most common approaches are those of R. H. BROOKS & A. T. COREY [9] and M. T. VAN GENUCHTEN [10]. According to the latter the relationship is described by

$$p_c = \frac{1}{\alpha_{vG}} [S_e^{(-1/m_{vG})} - 1]^{1/n_{vG}}, \quad (38)$$

where $\alpha_{vG}, m_{vG}, n_{vG}$ are parameters which depend on the type of the soil and S_e is the effective water saturation (37). For simplicity mostly it is assumed that $S_e \equiv S$. Obviously, the van Genuchten equation is a nonlinear relation in the saturation S .

It is clear that the relation (38) can be incorporated in a macroscopic model only under the condition that we know how to transfer true pressures p^{GR} and p^{FR} to the macroscopic

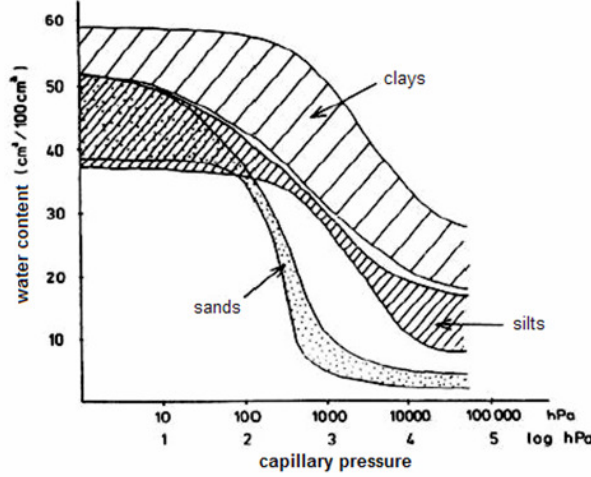


Figure 5: Range of capillary pressure curves for sands, silts and clays. Modified figure from HARTGE & HORN.

level. The simplest possibility is to use relations (32) but they may be too simplistic as we have already indicated. They may give reasonable results in linear models but, in general, the problem is still open.

4.3 Heat conduction

In Section 2 on one-component models the Fourier law (9) has been already introduced. It was mentioned that the coefficient λ , the heat conductivity, is for soils heavily dependent on the morphology and that this dependence cannot be reflected by a one-component model. Therefore we return here to this point in order to describe it by a three-component model. In Figure 6 (compare: FAROUKI [15]) we show an example of a nomogram in which the dependence of λ is demonstrated for various moisture contents (i.e. for various mass densities of the fluid if the mass density $\gamma_d = \rho^S$ of the skeleton is fixed), saturations (i.e. the volume fraction of the gas to the fluid component) and mass densities of the skeleton. As we see, in this example λ varies between 0.1 to 1.4 W/mK.

Unfortunately, the heat conductivity, λ , cannot be derived by means of any averaging procedure from microscopic conductivities of components. For this reason, we have to rely on empirical relations. These were proposed for soils some 30 years ago.

Recently developed experimental equipment such as TP O2 probe allow to make non-steady-state measurements of heat conductivity (e.g. A. GONTASZEWSKA [18]) for various morphologies of soils. In a work, published in 2008, SHAN XIONG CHEN [11] has proposed the following empirical relation for the conductivity

$$\lambda = \lambda_0^{1-n} \lambda_w^n [(1-b)S + b]^{cn}, \quad (39)$$

where S is the saturation, λ_0 is the grain heat conductivity, $\lambda_w = 0.61$ [W/mK] – heat conductivity of water and b, c are fitting parameters. For example, for sandy soils $\lambda_0 = 7.5$ [W/mK], $b = 0.0022$, $c = 0.78$. In an implicit way, this relation accounts as well for a dependence on temperature through λ_0 and λ_w .

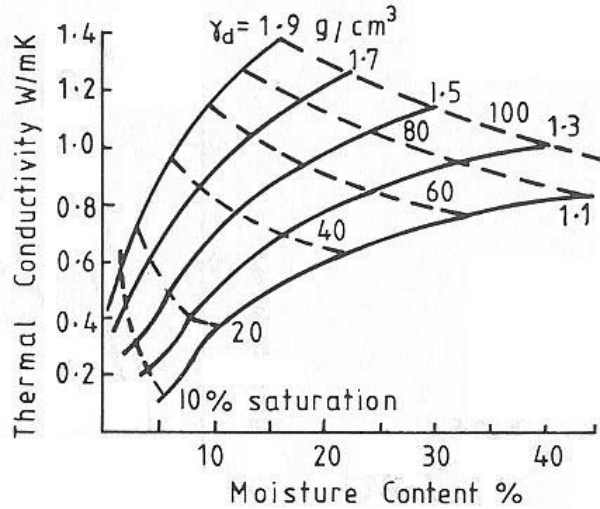


Figure 6: in dependence on the soil density, saturation and moisture content.

The results for the heat conductivity and corresponding theoretical one-component and multicomponent models play a particularly important role in description of freezing and frost heaving of soils.

Field equations for the above fields follow either from balance equations or from additional constitutive assumptions (saturation). Balance equations contain fluxes and sources and these, in turn, require additional information on material parameters.

5 Concluding remark

Three important issues of theoretical modeling should be mentioned.

The first one is the formulation of boundary conditions. In one-component models these are classical and extensively discussed in elasticity or plasticity. In multicomponent models the problem is more complicated because one has to formulate additional conditions for the extended set of partial differential equations. Even in the case of impermeable boundaries and such are phreatic surfaces of contact between saturated and dry domains of soils one has to formulate an equation of motion of the surface itself. Such moving boundaries yield the boundary value problems with free boundaries and these are usually ill-posed and create big mathematical problems. A physical presentation of this problem can be found in the book of BEAR [4]. The situation is even worse on permeable surfaces. A part of the conditions on such surfaces has been formulated by von Terzaghi who had shown that the external loading must be taken over by the whole stress vector $\mathbf{Tn} = \sigma_{ik}n_k$ where $\mathbf{n} = n_k\mathbf{e}_k$ is the unit normal vector of the boundary. The second condition was extensively discussed in the literature and it concerns the flow through the boundary. This boundary condition for inviscid fluids relates the pressure difference and the velocity of flow through the surface. It contains an additional material parameter, the so-called surface permeability. It plays a very important role on contact surfaces between different layers saturated with water and on the external surface which is the seepage face.

The second issue appears if the transition zones of not fully saturated soils appear. They are created, for example, by infiltration processes. In such processes one has to account for the capillary effects and an appropriate theoretical model must describe more than one fluid component. In the last few years such models are being developed. Presentation of a linear three-component model with capillary effects in applications to poroacoustics can be found in the book of B. ALBERS [1].

The third important issue is the development of software for geotechnical engineers which would account for all those theoretical problems which we have mentioned above. Such computational packages do not exist yet. It is only very recent that the research in this direction has been intensified. In particular, it concerns the formulation of some macroscopic constitutive laws in terms of microscopic material properties in which the micro-macro transition would be done in a numerical way. H. MATSSON, J. G. I. HELLSTRÖM, S. LUNDSTRÖM [28] formulate this problem in their extensive survey work from the year 2008 in the following manner: 'The main objective with this literature survey is to elucidate the state of the art of internal erosion in embankment dams in order to be able to formulate a research program for numerical modeling of internal erosion in a physically sound manner.'

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