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Iterative description of freezing and thawing processes in porous materials

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In memoriam Professor Olivier Coussy

The paper is devoted to the construction of an iterative procedure of calculation of mechanical properties of water saturated porous materials subject to freezing and thawing processes. The procedure consists of two steps. The first is the solution of equations for the linear poroelastic medium with material parameters depending on current porosity which measures the frost damage. The second one, based on the Gurson-Tvergaard-Needleman model of plastic deformations with porosity as a measure of damage, yields, among other quantities, the relation for porosity changes caused by freezing. Relations between material parameters and varying porosity follow from the micro-macro procedure which yields Gassmann and Epstein relations for compressibilities and permeability in the classical linear poroelasticity.

1 Introduction

Freezing processes in porous materials cause damage through large volume changes in the transition from water to ice, i.e. cryoswelling and through the high pressure of water in channels which does not freeze due to the shift of the melting point of ice and a flow of water from unfrozen regions. Water can come to the forming ice lenses at a temperature lower than the bulk freezing point owing to Gibbs-Thomson effect of confinement of water in pores. Damage caused by these freezing processes yields changes in the porosity and creation of microcracks in the skeleton. Subsequently, these microcracks coalesce and yield global destruction of the material. Typically, if diffusion processes can be neglected such deformations can be described by various models arising from the fundamental Gurson model (1977). The modern model in which the evolution of the damage parameter is identified with porosity changes was proposed by Tvergaard and Needleman (1981–1987) (see: [7, 10] for a detailed presentation). We present some details of this model and introduce its modification further in this paper.

Modeling of freezing and thawing is usually based on certain micromechanical considerations. It can follow from a model of microcracks in cavities (e.g. [8, 11]) and then diffusion processes are neglected or it may have a macroscopic character and then the skeleton is assumed to be elastic (e.g. [2] where an excellent analysis of different processes coupled to freezing is presented but the analysis is based on elastic potentials, or [15] where, apart from the erroneous nonlinear part, a construction of mass sources due to freezing for poroelastic materials is presented).

The simplified macroscopic model presented in this work is based on the continuous theory of immiscible mixtures. It is assumed that the porous material is fully saturated which means that we can use a two-component model. We distinguish two ranges of processes. The first, PE-range (poroelastic range), contains isothermal diffusive processes without freezing and it is modeled by a modified Biot theory presented in the next section. The second range, F-range (range of freezing) contains the processes of freezing and again, it is assumed that they are isothermal and diffusion-free. We describe them in Sect. 3 of the work. The whole long-term thermomechanical process of deformation, diffusion, freezing, and damage is assumed to be a sequence of such ranges. Therefore we call this approach an iterative description of freezing.

The procedure described above leaves out some important processes such as heat transfer or cryosuction in the range of freezing. However, it enables to solve some important engineering boundary value problems by simple and easily available

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numerical methods and standard packages such as ABAQUS or ANSYS. Within the range of damage the model is almost classical and within the range of a two-component diffusion it goes beyond known problems of porous media only due to heterogeneity of the material.

2 Modeling of the diffusion range (PE-range) without freezing

Continuous model of isothermal processes in a saturated porous material is described by the following fields

$$\left\{\rho^S, \rho^F, n, \mathbf{v}^S, \mathbf{v}^F, \mathbf{e}^S\right\}, \tag{1}$$

defined on a common domain $B \subset \Re^3$, where the first two quantities are partial mass densities of the skeleton and water, respectively, n denotes the current porosity, the next two fields are partial velocities, and the last one is the Almansi-Hamel deformation tensor. Linear model considered in this work satisfies the following conditions

$$\begin{aligned} &\left\|\mathbf{e}^S\right\| \ll 1, \quad |\varepsilon| \ll 1, \\ &\left(\mathbf{e}^S - \lambda^{(\alpha)}\mathbf{1}\right)\mathbf{n}^{(\alpha)} = \mathbf{0}, \quad \alpha = 1,2,3, \quad \left\|\mathbf{e}^S\right\| = \max\left|\lambda^{(\alpha)}\right|, \\ &\varepsilon = \frac{\rho_0^F - \rho^F}{\rho_0^F}, \end{aligned} \tag{2}$$

where eigenvalues $\lambda^{(\alpha)}$ are, obviously, principal stretches of the skeleton. They define volume changes of the skeleton for small deformations

$$e = \mathrm{tr}\,\mathbf{e}^S = \sum_{\alpha=1}^{3}\lambda^{(\alpha)}. \tag{3}$$

Field equations for those fields follow from partial balance laws of mass and momentum which for the linear theory have the following form

$$\begin{aligned} &\frac{\partial\rho^S}{\partial t} + \rho_0^S \mathrm{div}\,\mathbf{v}^S = 0, \quad \frac{\partial\rho^F}{\partial t} + \rho_0^F \mathrm{div}\,\mathbf{v}^F = 0, \quad \rho_0^S, \rho_0^F = \mathrm{const}, \\ &\rho_0^S\frac{\partial\mathbf{v}^S}{\partial t} = \mathrm{div}\mathbf{T}^S + \hat{\mathbf{p}}^S + \rho_0^S\mathbf{b}^S, \\ &\rho_0^F\frac{\partial\mathbf{v}^F}{\partial t} = \mathrm{div}\mathbf{T}^F + \hat{\mathbf{p}}^F + \rho_0^F\mathbf{b}^F, \end{aligned} \tag{4}$$

where all quantities with a zero lower index denote here and henceforth in this work their initial values. $\mathbf{T}^S, \mathbf{T}^F$ are partial Cauchy stress tensors and the momentum sources $\hat{\mathbf{p}}^S, \hat{\mathbf{p}}^F$ satisfy the conservation law

$$\hat{\mathbf{p}}^S = -\hat{\mathbf{p}}^F. \tag{5}$$

Body forces $\mathbf{b}^S, \mathbf{b}^F$ may contain noninertial forces.

The fluid component is assumed to be ideal which for water is rather well satisfied. Then

$$\mathbf{T}^F = \sigma_{kl}^F\mathbf{e}_k\otimes\mathbf{e}_l = -p^F\mathbf{1} \quad \text{i.e.} \quad \sigma_{\mathrm{kk}}^{\mathrm{F}} = -3p^F, \tag{6}$$

and $\mathbf{e}_k$ are unit base vectors of Cartesian coordinates. Obviously, in these coordinates

$$\mathbf{v}^S = v_i^S\mathbf{e}_i, \quad \mathbf{v}^F = v_i^F\mathbf{e}_i, \quad \mathbf{e}^S = e_{ij}^S\mathbf{e}_i\otimes\mathbf{e}_j, \tag{7}$$

and similarly for all other vector and tensor quantities in this work.

In addition, the nonequilibrium deviation of porosity from its equilibrium value n_E is assumed to satisfy the balance law [17]

$$\frac{\partial\Delta_n}{\partial t} + \Phi_0\mathrm{div}\left(\mathbf{v}^F - \mathbf{v}^S\right) = \hat{n}, \quad \Delta_n = n - n_E. \tag{8}$$

In this equation Φ_0 is a material parameter and its scalar character means that we assume isotropy of the medium.

The following quantities appearing in the balance laws must be specified by constitutive relations

$$\left\{\mathbf{T}^S, p^F, \hat{\mathbf{p}}^S, n_E, \hat{n}\right\}. \tag{9}$$

In the case of the simplest Biot-like model these constitutive quantities fulfill the following relations [1, 21]

$$\begin{aligned} &\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S e\mathbf{1} + 2\mu^S \mathbf{e}^S + Q\varepsilon\mathbf{1}, \quad p^F = p_0^F - Qe - \rho_0^F \kappa\,\varepsilon, \\ &\hat{\mathbf{p}}^S = -\hat{\mathbf{p}}^F = \pi\left(\mathbf{v}^F - \mathbf{v}^S\right), \quad \hat{n} = 0, \\ &n_E = n_0\left(1 + \delta\, e\right), \end{aligned} \tag{10}$$

where

$$\frac{\partial\,\mathbf{e}^S}{\partial\,t} = \text{sym grad}\,\mathbf{v}^S \quad \Rightarrow \quad \frac{\partial\,e}{\partial t} = \text{div}\,\mathbf{v}^S. \tag{11}$$

We have neglected an influence of the porosity gradient. This may be easily included [18, 22]. However, it would require an extension of micro-macro relations which is additionally needed and the solution of this problem cannot be presented in a closed form [18].

The model contains elastic constants of the skeleton λ^S, μ^S, the compressibility coefficient of water κ, the static Biot coupling coefficient Q, the material parameter for elastic changes of porosity δ, and the permeability coefficient π. The latter is a time independent scalar which means that we neglect hereditary effects connected with the diffusion and anisotropy effects connected with the tortuosity (comp. [22] for details).

An assumption of a vanishing source of porosity (i.e. $\hat{n} = 0$) means that with the above equations we model the range without freezing. In the next section we modify this assumption. In the present case the balance equation of porosity can be easily solved and we obtain the following relation for the current values of the porosity (e.g. [18])

$$n = n_0\left(1 + \delta\, e + \frac{\Phi_0}{n_0}\left(\varepsilon - e\right)\right). \tag{12}$$

We proceed to the specification of material parameters. One of them, the shear modulus μ^S cannot be derived from micro-macro transition relations. For this reason, it is either assumed to be given or, which may be the case for many soils, it is related to other material constants and to the Poisson ratio ν which is assumed to be a given constant (e.g. for soils $\nu \approx 0.33$).

In engineering applications it is more convenient to work with the bulk stress rather than with partial stresses. We make such a transformation for the purpose of further analysis. Addition of relations $(10)_1$ and $(10)_2$ yields

$$\begin{aligned} &\mathbf{T} = \mathbf{T}^S + \mathbf{T}^F = \mathbf{T}_0 + Ke\mathbf{1} + 2\mu^S \text{dev}\,\mathbf{e}^S - C\zeta\mathbf{1}, \quad \text{dev}\,\mathbf{e}^S = \mathbf{e}^S - \tfrac{1}{3}\text{tr}\,\mathbf{e}^S\mathbf{1}, \\ &p = \frac{p^F}{n_0} = p_0 - Ce + M\zeta, \end{aligned} \tag{13}$$

where $\mathbf{T}_0, p_0$ are the initial bulk stress and the initial pore pressure, respectively, p is called the pore pressure and the so-called Biot material parameters K, C, M are defined by the relations

$$K = \lambda^S + \tfrac{2}{3}\mu^S + \rho_0^F\kappa + 2Q, \quad C = \frac{Q + \rho_0^F\kappa}{n_0}, \quad M = \frac{\rho_0^F\kappa}{n_0^2}. \tag{14}$$

They replace the parameters λ^S, Q, κ. The new field, ζ, is called the increment of fluid content and it is defined by the relation

$$\zeta = n_0\left(e - \varepsilon\right). \tag{15}$$

It is convenient to work with this variable because, according to mass balance equations, it satisfies the relation

$$\frac{\partial\,\zeta}{\partial\,t} = \text{div}\left(\mathbf{v}^F - \mathbf{v}^S\right), \tag{16}$$

which means that it is identically zero when the diffusion does not arise: $\mathbf{v}^S = \mathbf{v}^F$. It yields the following form of the porosity balance equation (8)

$$\frac{\partial}{\partial t}\left(\Delta_n - \frac{\Phi_0}{n_0}\zeta\right) = \hat{n}\,. \tag{17}$$

Consequently, the balance equation of porosity reduces in this linear model to a pure evolution equation and this means, in turn, that it does not require boundary conditions.

For completeness we write the transformed balance equations of momentum in which we neglect inertial forces. These are immaterial in processes of freezing. We have

$$\begin{aligned} &\operatorname{div}\mathbf{T} + \rho_0\,\mathbf{b} = 0, \quad \rho_0 = \rho_0^S + \rho_0^F, \quad \rho_0\,\mathbf{b} = \rho_0^S\mathbf{b}^S + \rho_0^F\mathbf{b}^F, \\ &\text{grad}\,(n_0 p) + \pi\left(\mathbf{v}^F - \mathbf{v}^S\right) - \rho_0^F\mathbf{b}^F = 0, \end{aligned} \tag{18}$$

and the deformation measure $\mathbf{e}^S$ must satisfy the following compatibility condition

$$\frac{\partial\,\mathbf{e}^S}{\partial t} = \text{sym grad}\,\mathbf{v}^S. \tag{19}$$

Eq. $(18)_2$ describes a paradox related to the porosity n_0 varying in space. Clearly, for absent body forces and constant pore pressure a sheer variation of porosity, which is, obviously, a geometrical property, creates a difference in velocities of the components. This hydrostatic paradox has been recently discussed by El Tani [4]. It can be removed by the assumption that the momentum source depends on the porosity gradient [19] which we ignore in this paper. Strictly speaking, one would have to correct the constitutive relation $(13)_2$ by an addition of a term proportional to the increment of the porosity.

The set (13), (16), (17), (18), and (19) is the full set of field equations. Sometimes, it is convenient to replace the increment of fluid contents by the pore pressure as an unknown field but we shall not do so in this work.

We return now to the estimation of Biot's material parameters (14). This can be done by means of the micro-macro transition [18]. In such an approach it is assumed that material parameters of components, i.e. the material properties of the system for $n_0 = 0$ and $n_0 = 1$, respectively, are known. As a result of the so-called gedankenexperiments we obtain for the case under consideration the Gassmann relations (e.g. see [18])

$$\begin{aligned} &K = \frac{\left(K_s - K_d\right)^2}{\frac{K_s^2}{K_W} - K_d} + K_d, \quad C = \frac{K_s\left(K_s - K_d\right)}{\frac{K_s^2}{K_W} - K_d}, \quad M = \frac{K_s^2}{\frac{K_s^2}{K_W} - K_d}, \\ &K_W = \left(\frac{1-n_0}{K_s} + \frac{n_0}{K_f}\right)^{-1}, \end{aligned} \tag{20}$$

where K_s, K_f, K_d are the compressibility module of the skeleton, of water, and the so-called drained compressibility modulus, respectively. These relations specify the dependence of macroscopic parameters on the initial porosity n_0.

The same procedure yields relations for the parameters δ, Φ_0 of the porosity balance equation (comp. [20]). As we do not need them in this paper we shall not quote them.

There remains the specification of the permeability coefficient π. We assume that the random structure of the porous material yields isotropy which means that this material parameter is a scalar. In 1927 Kozeny proposed a relation for this parameter, connecting it with the viscosity of the fluid in channels [9]. However, in this old paper the dependence on the tortuosity was not correct and it has been reformulated by Epstein [5] in 1989. Tortuosity τ is in the isotropic case a quantity, which is equal to the fraction of the length of streamlines between two points lying close to each other to the distance of these points. Consequently, $\tau \geq 1$. Epstein's proposition for the corrected Kozeny relation has the form

$$\pi = \frac{9}{4}\frac{\gamma\,b\mu_v}{d^2}\tau^2\left(\frac{1-n_0}{n_0}\right)^2, \tag{21}$$

where

$$\gamma = \frac{g\rho_0^F}{n_0}, \tag{22}$$

g is the gravity acceleration, b is the so-called capillary shape factor (e.g. 32 for circular pores and 48 for parallel slits), μ_v is the true dynamic viscosity of the fluid, and d the characteristic diameter. The coefficient $9b\tau^2/4$ is often assumed to be equal to 180. For $b = 33.3$ it corresponds to the tortuosity $\tau = 1.56 \approx \pi/2$. For a detailed discussion of permeability we refer to paper [22].

3 Governing equations for material damaged by freezing (F-range)

Creation of ice lenses in porous materials yields a number of accompanying effects which substantially change the properties of the medium. First, in the domain $\mathcal{B}_f \subset \mathcal{B} \subset \Re^3$ of freezing diffusion is hanpered. This means that the increment of the fluid content ζ in $\mathcal{B}_f$ is identically zero in our model. Hence, we can use a one-component, composite-like, description of the medium.

Inhibition of diffusion and the volume change of water through freezing raise the pore pressure to very high values. This has been found in early experiments on heaving of soils (e.g. [13]) and was confirmed in numerous modern experiments (e.g. [14]). In a few recent works O. Coussy and coworkers proposed a theoretical description of freezing and thawing processes in porous materials which yield relations for the pore pressure and the shift of the melting points [2,3,6]. It is based on a microscopic approach in which freezing processes in pores are described by the classical thermodynamic equations of Gibbs-Thomson and Young-Laplace on a scale of a simple pore structure (spherical voids with cylindrical connections). Freezing and thawing are described as slow time evolution processes in which pore pressure changes according to the evolution equation

$$\tau \frac{dp}{dt} + p = S_f \left(T_f - T \right) , \tag{23}$$

where $\tau = \mu_v \left(3/4\mu^S + 1/K_{\text{ice}} \right)$ is the relaxation time and K_{ice} is the compressibility modulus of ice [3]. In addition, T_f is a uniform initial temperature and S_f is the entropy of fusion. For the purpose of the iterative method presented in this work this evolution process is neglected. This is justified for relatively slow capillary motion of water as the typical relaxation time is very small indeed. For frozen soils (e.g. Cairo sand [16]) we have the following typical values $\mu^S = 161$ MPa, $\text{K}_{\text{ice}} = 1200$ MPa, and the viscosity of water under normal conditions is $\mu_{\text{v}} = 1.002 \times 10^{-3}$ Pa s. These yield the following order of magnitude of the relaxation time: $\tau = 0.55 \times 10^{-11}$ s.

Consequently, we can consider processes in each range of freezing as rate independent quasistatic processes of damage. In such processes cavities are nucleated, grow by plastic deformation and yield mesocracks by coalescence. These three mechanisms, characteristic for geomaterials, are described by the extension of Gurson's model proposed by Tvergaard and Needleman [10, 12]. According to the Gurson-Tvergaard-Needleman (GTN) model there exists a critical value of porosity n_C, created by the microdamage, at which microcracks coalesce and yield a macroscopic damage of the material. This would be the last F-range in our iteration process beyond which the model does not work. Hence the porosity considered in this work must be smaller than this critical value. Then the GTN model is based on the following yield function [10]

$$f_{GTN} = \frac{\sigma_{eq}^2}{\sigma_s^2} + 2q_1 n \cosh \left(\frac{1}{2} q_2 \frac{\sigma_{kk}}{\sigma_s} \right) - 1 - (q_1 n)^2 , \tag{24}$$

where

$$\sigma_{eq} = \sqrt{\tfrac{3}{2} \sigma_{ij}^D \sigma_{ij}^D}, \quad \sigma_{ij}^D = \sigma_{ij} - \tfrac{1}{3} \sigma_{kk} \delta_{ij}, \tag{25}$$

q_1, q_2 are material parameters which for metals are approximately 1.5 and 1.0, respectively. These values seem to be confirmed by a numerical analysis of the so-called cell model of damage [7]. Most likely the same values can be also accepted for other materials. n denotes the current porosity. σ_s is the yield stress and it may consist of two parts: σ_Y and σ_H with $\sigma_s = \sigma_Y + \sigma_H$, where these contributions are the yield stress at the beginning of the damage process and the isotropic hardening, respectively. As already mentioned, Hao and Brocks [7] justified the above yield function by averaging the microscopic cell model over a simple Representative Elementary Volume.

The GTN model is based on the normality rule which yields the following relation for plastic strain rate

$$\dot{e}_{ij}^p = \dot{\lambda} \left[\frac{3\sigma_{ij}^D}{\sigma_s^2} + \frac{q_1 q_2 n}{\sigma_s} \sinh \left(\tfrac{1}{2} q_2 \frac{\sigma_{kk}}{\sigma_s} \right) \delta_{ij} \right], \tag{26}$$

where, as usual, the plastic multiplier $\dot{\lambda}$ follows from the consistency conditions at yield: $f_{GTN} = 0, \; \dot{f}_{GNT} = 0$. In contrast to classical plasticity theory, the rate of equivalent plastic strain is defined by the relation

$$\dot{e}_{eq}^p = \frac{\sigma_{ij} \dot{e}_{ij}^p}{(1 - n) \, \sigma_s}. \tag{27}$$

The most important issue of damage by freezing is the evolution of porosity due to damage. As changes of the damage parameter and of the porosity are assumed to be identical we can write for the source of porosity in the porosity balance equation (8)

$$\hat{n} = (1 - n) \; \dot{e}_{kk}^p + \dot{n}_n, \tag{28}$$

where the first part is, obviously, the contribution of growth of voids due to plastic deformation while the second contribution describes nucleation. This process is controlled by the growth of the ice lenses and, therefore it seems to be reasonable to assume that the rate is proportional to the equivalent plastic strain rate

$$\dot{n}_n = N_n \dot{e}^p_{eq}, \tag{29}$$

where N_n is a material parameter dependent on the latent heat and the temperature of transformation.

Now, bearing in mind the assumption that in the range of freezing there is no diffusion we can write the porosity balance equation (8) in the following form

$$\frac{\partial n}{\partial t} = n_{in}\delta \frac{\partial e}{\partial t} + (1-n)\,\frac{\partial e^p_{kk}}{\partial t} + N_n \frac{\partial e^p_{eq}}{\partial t}, \tag{30}$$

where n_{in} replaces the initial porosity n_0 of the first PE-range and it is the value of porosity in the PE-range preceding the F-range under consideration. It is simultaneously the initial value for the above evolution equation.

Lack of diffusion yields easily the relation between elastic strain and the bulk stress. This relation is needed if we want to formulate equations analogous to the Prandtl-Reuss equations of classical plasticity. Namely, according to (13) for $\zeta = 0$ we have

$$e_{el} = \mathrm{tr}\,\mathbf{e}^S_{el} = \frac{1}{3K}\mathrm{tr}\left(\mathbf{T} - \mathbf{T}_0\right), \quad \mathrm{dev}\,\mathbf{e}^S_{el} = \frac{1}{2\mu^S}\mathrm{dev}\,\left(\mathbf{T} - \mathbf{T}_0\right), \tag{31}$$

where $\mathbf{e}_{el}$ is an elastic part of the deformation, $\dot{\mathbf{e}}^S = \dot{\mathbf{e}}^S_{el} + \dot{\mathbf{e}}^S_p, \quad \mathbf{e}^S_p = e^p_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$. The material parameters are given by the relations

$$K = \frac{\left(K_s - K_d\right)^2}{\frac{K_s^2}{K_W} - K_d} + K_d, \quad K_W = \left(\frac{1-n}{K_s} + \frac{n}{K_f}\right)^{-1}, \quad \mu^S = \frac{3}{2}\frac{1-2\nu}{1+\nu}K, \tag{32}$$

in which ν is a constant Poisson number and n denotes the current value of porosity. These relations replace the classical relations between material parameters and the damage parameter [10].

The set of Eqs. (24)–(30) together with the equilibrium condition for bulk stresses $(18)_1$ forms the complete set of field equation for the F-range. As there is no diffusion in the domain $\mathcal{B}_f$ boundary conditions are simple because they do not have to describe flows through the boundary of this domain which would be the case for the two-component model with diffusion.

4 Iterative procedure

In this short note we do not present any numerical solutions of boundary value problems. However, we would like to indicate some properties of the model presented in the previous two sections in relation to the dependence of field equations on the damage parameter. The procedure begins with the solution of a boundary value problem for the two-component poroelastic material (PE-range) as described in Sect. 2. It is assumed that the temperature T in this range is higher than the freezing temperature, T_f, and the initial porosity n_0 is assumed to be constant in time. Among the results of the calculations one obtains the field $n = n_{(1)}\,(\mathbf{x})$ described by relation (12). As the model is linear all material parameters are evaluated for the initial porosity n_0. They are given by relations (20) and (21). In the second step it is assumed that the temperature is lowered to a value smaller than the temperature of freezing, T_f. Solution of the corresponding boundary value problem in the F-range is made by means of equations presented in Sect. 3 with the initial porosity equal to the porosity $n_{(1)}\,(\mathbf{x})$ of the preceding PE-range. Among the other results new values of porosity $n = n_{(2)}\,(\mathbf{x})$ follow from these calculations as solution of Eq. (30). This field measures the damage of the material in the F-range due to freezing. Now, we may proceed to the next step which is again the PE-range for the new value of the temperature, higher than the temperature of freezing. The initial value of the porosity is now $n_{(2)}\,(\mathbf{x})$ which means that, in contrast to the first PE-range, the boundary value problem in this range is heterogeneous. New values of material parameters calculated by means of relations (20) and (21) are now dependent on the position **x**. We continue this iteration until, after a certain number of freezing-thawing cycles the porosity exceeds at some place a critical value (coalescence of microcracks!) and the material is treated as destroyed.

Let us note that material parameters depend in this iteration procedure on the damage in a different way in both ranges. Elastic parameters such as the bulk compressibility K are at any given point **x** constant within one PE-range but they change in the process of loading in the F-range. This is different from standard damage mechanics.

In the next two figures we present an example of dependence of two material parameters on the porosity. We use the following data

$$K_s = 48 \times 10^9 \text{ Pa}, \quad K_f = 2.25 \times 10^9 \text{ Pa}, \quad K_d = \frac{K_s}{1 + 50n}, \quad \pi(n = 0.1) = 10^{10} \, \frac{\text{kg}}{\text{m}^3\text{s}}. \tag{33}$$

The relation for the drained modulus K_d, the so-called Geertsma formula, is often used in models of soils.

In Fig. 1 we present the compressibility modulus K. It is plotted for a wide range of porosity from $n = 0.1$ to $n = 0.5$ in order to show the shape of the function. The range 0.3–0.35 is plotted as the solid line because this is approximately the range of changes of porosity due to damage if the initial porosity is chosen to be $n_0 = 0.3$. Clearly, the compressibility modulus is decaying in a similar manner as for the classical laws of damage mechanics.

In Fig. 2 we show the dependence of the permeability coefficient π on porosity. We have chosen $\pi = 10^{10}\,\text{kg/m}^2\text{s}$ for $n = 0.1$ which corresponds to the app. value 0.01 darcy $(= 10^{-14}\,\text{m}^2)$ of the so-called intrinsic permeability (e.g. [22]). Again, the coefficient is plotted for a wide range of porosity in order to show the shape of the function. In the range 0.3–0.35 it is drawn as a solid line. As expected, the damage-increasing porosity yields a decreasing permeability coefficient which means that the diffusive force – a resistance to the relative motion – decays with growing damage.

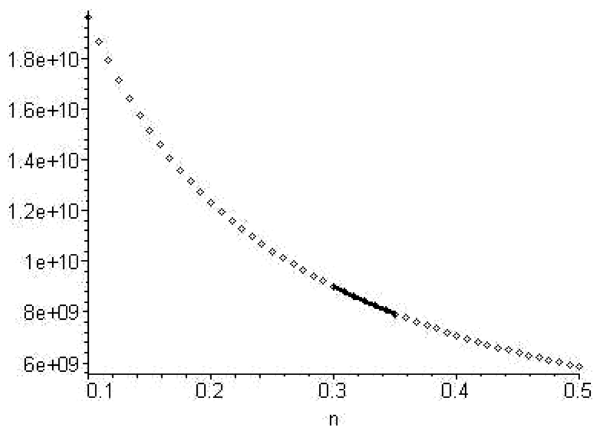

Fig. 1 Compressibility modulus K as a function of porosity.

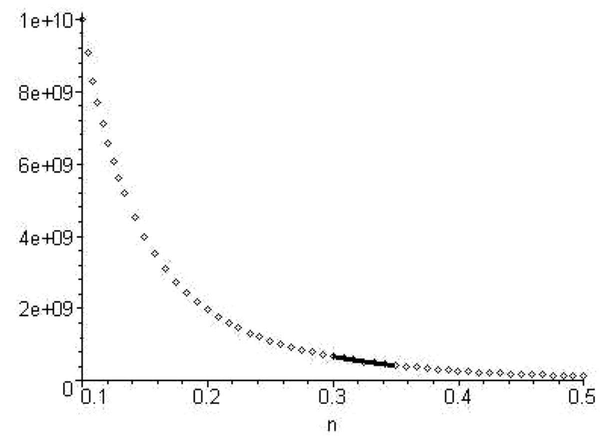

Fig. 2 Permeability coefficient π as a function of porosity.

This example confirms the expectation that micro-macro relations of mechanics of porous materials may replace speculative dependencies on the damage parameter within classical damage mechanics.

We do not present in this short note other numerical results as they require the application of complex numerical software.

5 Concluding remarks

The iterative description of freezing processes of porous materials proposed in this work seems to be sufficiently simple for practical applications in such engineering problems as frost heaving of soils or freezing techniques in construction of tunnels and pits. It is also capable of extensions on nonisothermal processes with diffusion in the range of freezing. In particular this latter problem seems to be important from the physical point of view. Damage done by ice lenses appears primarily due to the increment of pore pressure which in turn is a consequence of cryosuction, i.e. diffusive motion in the medium. However, from the point of view of numerical evaluation such extensions may be very difficult.

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