Threshold of liquefaction due to weakly nonlinear acoustic waves in a poroelastic medium

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Liquefaction
and other ground instabilities

Ground rupture (Taiwan)

Landslide in El Salvador (Colonia Las Colinas)
by the earthquake 13.01.2001

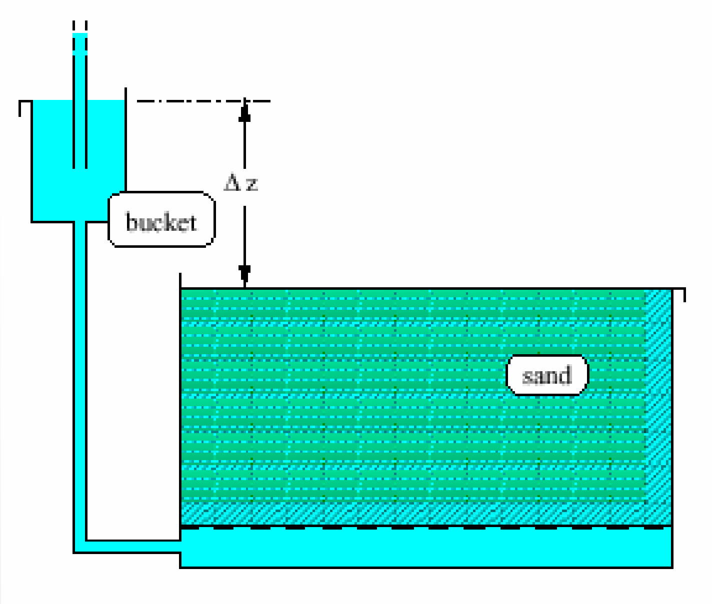
Liquefaction after
Niigata earthquake
Japan 1964

Tilt due to liquefaction
(Adapazari)

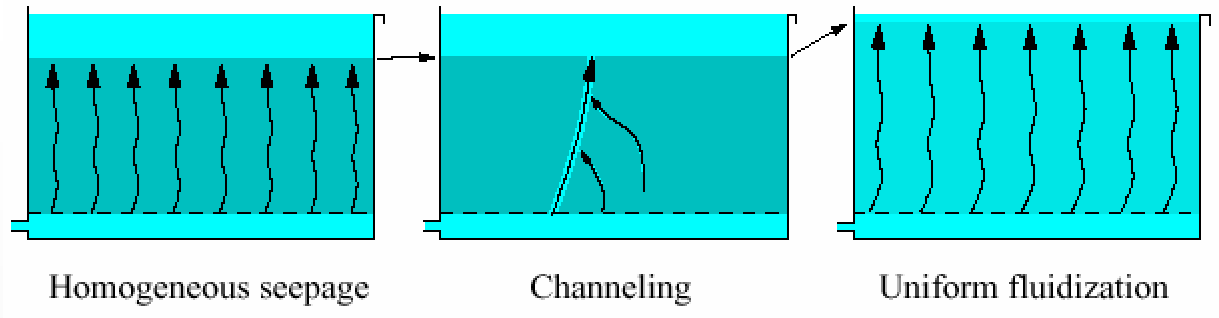
Experiments on a saturated sand

PhD-Thesis: Theo Wilhelm, University of Innsbruck, 2000

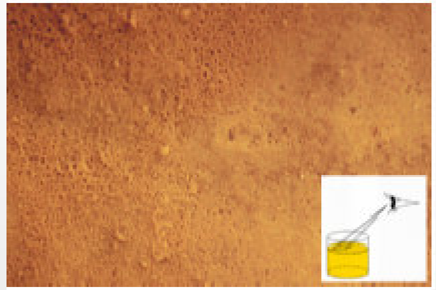
Theo Wilhelm, K. Wilmanski; *On the Onset of Flow Instabilities in Granular Media due to Porosity Inhomogeneities, Int. J. Multiphase Flows,* 28, 1929-1944, 2002.



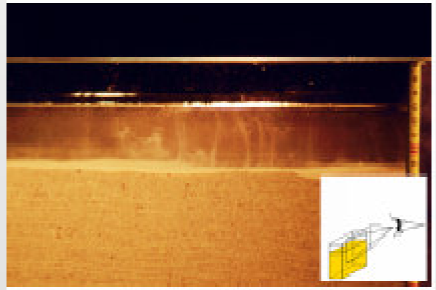
Experimental setup



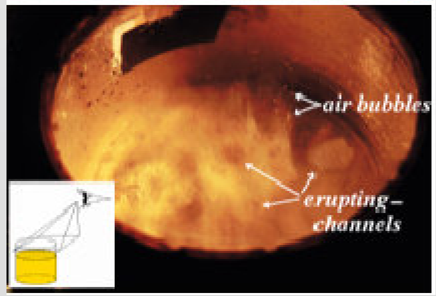
Flow regimes in sand-water mixtures under seepage conditions



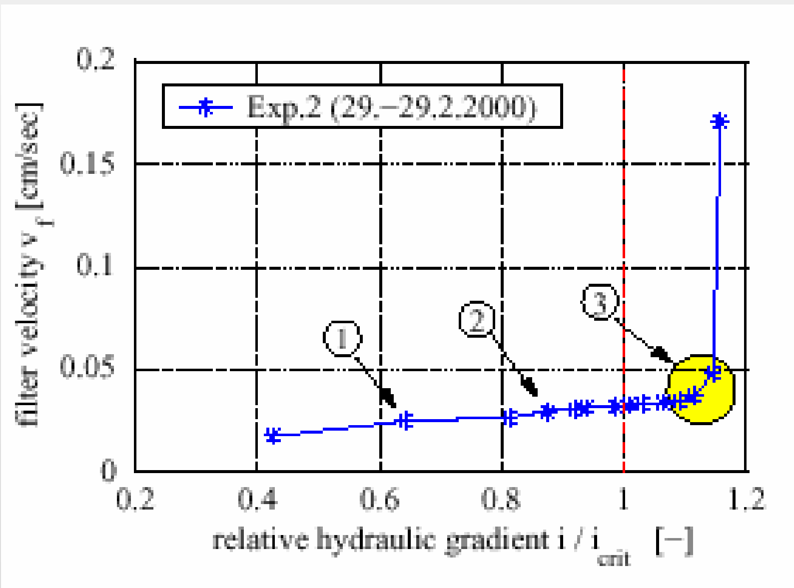
1. Homogeneously distributed micro-channels (small dark spots) on the top surface of a sand speciment subject to seepage. Diameters of channels up to 1 mm.



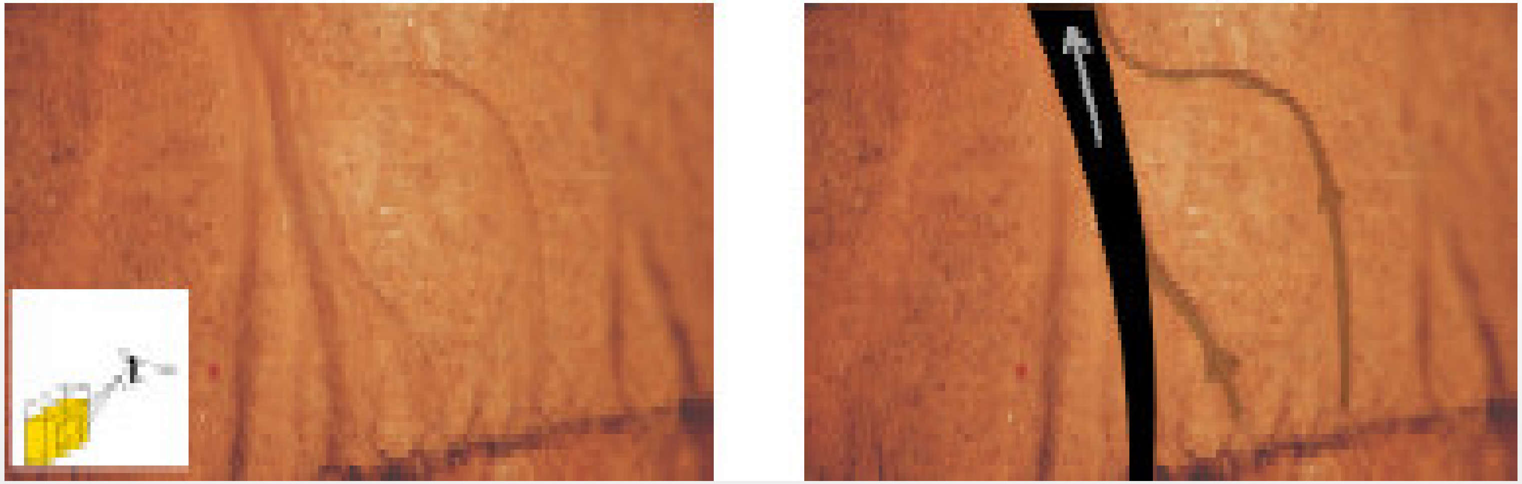
2. Channels with diameters up to several mm have formed. Very fine particles flushed out through them are visible in the water layer above the sand surface.



3. Instabilities (washed out air bubbles, erupting channels) shortly before the eruption of a main channel.



Experimental data from a seepage experiment.



A main channel (indicated by the dark shadow in the right figure) is fed by two smaller channels (indicated by the light grey areas in the right figure). Both smaller channels changed their direction due to the attraction of the main channel

Two-component weakly nonlinear model of poroelastic saturated media

Fields: $(\mathbf{x},t) \to \{\rho^S, \rho^F, n, \mathbf{v}^S, \mathbf{v}^F, \mathbf{e}^S\}$.

ρ^S - partial mass density of the skeleton,

ρ^F - partial density of the fluid,

n - porosity (volume fraction of the fluid in REV),

$\mathbf{v}^S$ - velocity of the skeleton,

$\mathbf{v}^F$ - velocity of the fluid,

$\mathbf{e}^S$ - Almansi-Hamel deformation tensor of the skeleton.

$$\varepsilon = \frac{\rho_0^F - \rho^F}{\rho_0^F}$$ -volume changes of the fluid,

K. Wilmanski; *Critical time for acoustic waves in weakly nonlinear poroelastic materials, Continuum Mech. Thermodyn.*, **17**, 2, 171-181, 2005.

Second order approximation:

$$\max\left\{\sum_{i,j}\left|\lambda_e^{(i)}\lambda_e^{(j)}\right|\right\} << 1, \quad \det\left(\mathbf{e}^S - \lambda_e^{(i)}\mathbf{1}\right) = 0,$$

$$|\varepsilon| << 1.$$

Equilibrium porosity: $n \approx n_E = n_0\left(1+\delta\, tr\,\mathbf{e}^S\right) \equiv n_0\left(1+\delta I\right).$

Mass density of the skeleton: $\rho^S = \rho_0^S\left(1 - I - \frac{1}{2}\left(I^2 + 4II\right)\right), \quad I = tr\mathbf{e}^S, \quad II = \frac{1}{2}\left(I^2 - tr\mathbf{e}^{S2}\right).$

Remaining Field Equations:

$$\frac{\partial \varepsilon}{\partial t} + div\left(\varepsilon - 1\right)\mathbf{v}^F = 0,$$

$$\rho^S\left(\frac{\partial \mathbf{v}^S}{\partial t} + \mathbf{L}^S\mathbf{v}^S\right) = div\,\mathbf{T}^S + \pi\left(\mathbf{v}^F - \mathbf{v}^S\right),$$

$$\rho^F\left(\frac{\partial \mathbf{v}^F}{\partial t} + \mathbf{L}^F\mathbf{v}^F\right) = -grad\, p^F - \pi\left(\mathbf{v}^F - \mathbf{v}^S\right),$$

$$\mathbf{L}^S = grad\,\mathbf{v}^S, \quad \mathbf{L}^F = grad\,\mathbf{v}^F.$$

Integrability condition:

$$\frac{\partial \mathbf{e}^S}{\partial t} + \mathbf{v}^S \cdot grad\,\mathbf{e}^S = \frac{1}{2}\left(\mathbf{L}^S + \mathbf{L}^{ST}\right) - \left(\mathbf{L}^{ST}\mathbf{e}^S + \mathbf{e}^S\mathbf{L}^S\right).$$

π – constant within the second order model

Constitutive relations (Signorini-like): $\mathbf{T}^S = \mathbf{T}_0^S + \left(\lambda^S I + \frac{1}{2}\left(\lambda^S + \mu^S\right)I^2\right)\mathbf{1} + 2\left(\mu^S - \left(\lambda^S + \mu^S\right)I\right)\mathbf{e}^S$

Solid:

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda_0^S I\mathbf{1} + 2\mu_0^S\mathbf{e}^S + \left(\delta\left.\frac{\partial \lambda^S}{\partial n}\right|_0 n_0 I^2 + \frac{1}{2}\left(\lambda_0^S + \mu_0^S\right)I^2\right)\mathbf{1} + 2\left(\delta\left.\frac{\partial \mu^S}{\partial n}\right|_0 n_0 I - \left(\lambda_0^S + \mu_0^S\right)I\right)\mathbf{e}^S,$$

Fluid:

$$p^F = p_0^F - \rho_0^F\kappa_0\varepsilon - \rho_0^F\delta\left.\frac{\partial \kappa}{\partial n}\right|_0 n_0 I\varepsilon.$$

1D Model

$$\mathbf{v}^S = v^S \mathbf{e}_x, \quad \mathbf{v}^F = v^F \mathbf{e}_x, \quad \mathbf{e}^S = e^S \mathbf{e}_x \otimes \mathbf{e}_x, \quad |\mathbf{e}_x| = 1.$$

Hence

$$I = e^S, \quad II = 0, \quad \mathbf{L}^S = \frac{\partial v^S}{\partial x} \mathbf{e}_x \otimes \mathbf{e}_x, \quad \mathbf{L}^F = \frac{\partial v^F}{\partial x} \mathbf{e}_x \otimes \mathbf{e}_x.$$

$$\rho^S = \rho_0^S \left(1 - e^S - \frac{1}{2} e^{S2} \right), \quad \rho^F = \rho_0^F (1 - \varepsilon), \quad n = n_0 \left(1 + \delta e^S\right).$$

Partial stresses:

$$\sigma^S = \sigma_0^S + \left(\lambda^S + 2\mu^S\right) e^S - \frac{3}{2}\left(\lambda_0^S + \mu_0^S\right) e^{S2}, \quad p^F = p_0^F - \rho_0^F \kappa \varepsilon,$$

$$\lambda^S + 2\mu^S = \lambda_0^S + 2\mu_0^S + \delta n_0 \frac{\partial}{\partial n}\left(\lambda^S + 2\mu^S\right)\Big|_0 e^S.$$

$$\kappa = \kappa_0 + \delta n_0 \frac{\partial \kappa}{\partial n}\Big|_0 e^S.$$

Governing set of equations:

$$\frac{\partial u'_A}{\partial t'} + A'_{AB}\frac{\partial u'_B}{\partial x'} = B'_A, \quad t' = \frac{t\pi}{2\rho_0^S}, \quad x' = \frac{x\pi}{2\rho_0^S c_{P1}},$$

$$c_{P1}^2 = \frac{\lambda_0^S + 2\mu_0^S}{\rho_0^S}, \quad c_S^2 = \frac{\mu_0^S}{\rho_0^S}, \quad c_{P2}^2 = \kappa_0, \quad c_s = \frac{c_S}{c_{P1}}, \quad c_f = \frac{c_{P2}}{c_{p1}},$$

Auxiliary quantities:

$$l^{F}{}' = \delta n_0 \frac{\partial}{\partial n}\frac{\rho_0^S \kappa}{\lambda_0^S + 2\mu_0^S}\bigg|_0,$$

$$l^{S}{}' = 2\delta n_0 \frac{\partial}{\partial n}\frac{\lambda^S + 2\mu^S}{\lambda_0^S + 2\mu_0^S}\bigg|_0 - \left(2 - 3c_s^2\right).$$

$$[u'_A]^T = \left[\varepsilon, v^F{}', v^S{}', e^S\right]^T, \quad v^F{}' = \frac{v^F}{c_{P1}}, \quad v^S{}' = \frac{v^S}{c_{P1}},$$

★

$$[A'_{AB}] = \begin{bmatrix} v^F{}' & \varepsilon - 1 & 0 & 0 \\ -c_f^2(1+\varepsilon) - l^F{}' e^S & v^F{}' & 0 & -l^F{}'\varepsilon \\ 0 & 0 & v^S{}' & -1 - l^S{}' e^S \\ 0 & 0 & -\left(1 - 2e^S\right) & v^S{}' \end{bmatrix},$$

$$[B'_A]^T = \left[0, -2(1+\varepsilon)\left(v^F{}' - v^S{}'\right), 2\left(1 + e^S\right)\left(v^F{}' - v^S{}'\right), 0\right]^T.$$

Evolution of the amplitude of weak discontinuity

Wave front $\mathcal{S}$

$$[[u'_A]] = (u'_A)^+ - (u'_A)^-, \quad \left[\left[\frac{\partial u'_A}{\partial t'}\right]\right] = -c\left[\left[\frac{\partial u'_A}{\partial x'}\right]\right],$$

$$\left[\left[\frac{\partial^2 u'_A}{\partial t'\partial x'}\right]\right] = \frac{d}{dt'}\left[\left[\frac{\partial u'_A}{\partial x'}\right]\right] - c\left[\left[\frac{\partial^2 u'_A}{\partial x'^2}\right]\right], \quad etc.,$$

$$(A'_{AB} - c\delta_{AB})\left[\left[\frac{\partial u'_A}{\partial x'}\right]\right] = 0 \quad \Rightarrow \quad \left[\left[\frac{\partial u'_A}{\partial x'}\right]\right] = \mathcal{A} r'_A, \quad r'_A r'_A = 1.$$

Evolution of the amplitude

$$\frac{d\mathcal{A}}{dt'} + \alpha'_1 \mathcal{A} + \alpha'_2 \mathcal{A}^2 = 0 \quad \Rightarrow$$

$$\Rightarrow \quad \frac{1}{\mathcal{A}} = \left[\frac{1}{\mathcal{A}_0} + \int_0^{t'} \alpha'_2 \exp\left(-\int_0^{\eta} \alpha'_1 \, ds\right) d\eta\right] \exp\left(\int_0^{t'} \alpha'_1 \, ds\right).$$

Critical time

$$\left[\frac{1}{\mathcal{A}_0} + \int_0^{t'_c} \alpha'_2 \exp\left(-\int_0^{\eta} \alpha'_1 \, ds\right) d\eta\right] = 0.$$

★ Matrix A'_{AB} on the positive side of the P1-front – solution of the eigenvalue problem

Eigen-value	Right eigenvector r'_A	Left eigenvector l'_A
+1	$[0,0,-1/\sqrt{2},1/\sqrt{2}]$	$[0,0,-1/\sqrt{2},1/\sqrt{2}]$
-1	$[0,0,1/\sqrt{2},1/\sqrt{2}]$	$[0,0,1/\sqrt{2},1/\sqrt{2}]$
c_f	$[1/\sqrt{(1+c_f^2)},-c_f/\sqrt{(1+c_f^2)},0,0]$	$[c_f/\sqrt{(1+c_f^2)},-1/\sqrt{(1+c_f^2)},0,0]$
$-c_f$	$[1/\sqrt{(1+c_f^2)},c_f/\sqrt{(1+c_f^2)},0,0]$	$[c_f/\sqrt{(1+c_f^2)},1/\sqrt{(1+c_f^2)},0,0]$

Coefficients in the equation for the amplitude – P1-characteristic:

$$\alpha'^{(1)}_{1} = -l'_A \frac{\partial B'_A}{\partial u'_C} r'_C \frac{1}{r'_D l'_D}\bigg|^{(1)} = 1,$$

$$\alpha'^{(1)}_{2} = -l'_A \frac{\partial A'_{AB}}{\partial u'_C} r'_B r'_C \frac{1}{r'_D l'_D}\bigg|^{(1)} = -\frac{1}{\sqrt{2}}\left(2 - \tfrac{1}{2} l^{S}{}'\right).$$

Solution:

$$\mathcal{A} = e^{-t'}\left[\frac{1}{\mathcal{A}_0} - \frac{1}{\sqrt{2}}\left(2 - \tfrac{1}{2} l^{S}{}'\right)\left(1 - e^{-t'}\right)\right]^{-1}.$$

Critical time:

$$t'_c = -\ln\left[1 - \frac{\sqrt{2}}{\mathcal{A}_0\left(2 - \tfrac{1}{2} l^{S}{}'\right)}\right].$$

Threshold amplitude

$$\mathcal{A}_0 > \frac{\sqrt{2}}{2 - \tfrac{1}{2} l^{S}{}'}.$$

Existence of critical time: $\mathcal{A}_0\left(2 - \tfrac{1}{2} l^{S}{}'\right) > 0.$

Physical amplitudes:

$$\left[\left[\frac{\partial e^S}{\partial x'}\right]\right] = -\left.\frac{\partial e^S}{\partial x'}\right|^- = \mathcal{A} r'^{(1)}_4 = \tfrac{1}{\sqrt{2}}\mathcal{A} \quad \Rightarrow \quad \left.\frac{\partial e^S}{\partial x}\right|^- = -\frac{\pi}{2\sqrt{2}\rho_0^S c_{P1}}\mathcal{A}.$$

$$\left[\left[\frac{\partial v'^S}{\partial x'}\right]\right] = -\left.\frac{\partial v'^S}{\partial x'}\right|^- = \mathcal{A} r'^{(1)}_3 = -\tfrac{1}{\sqrt{2}}\mathcal{A} \quad \Rightarrow \quad \left.\frac{\partial e^S}{\partial t}\right|^- = \frac{\pi}{2\sqrt{2}\rho_0^S}\mathcal{A}.$$

Numerical example:

$$\pi = 10^7 \tfrac{kg}{m^3 s} \quad (app.\, 0.1\, Darcy), \quad \rho_0^S = 2500 \tfrac{kg}{m^3}, \quad c_{P1} = 2500 \tfrac{m}{s},$$

$$\mathcal{A} = 0.4 \quad \Rightarrow \quad \left.\frac{\partial e^S}{\partial x}\right|^- \approx 0.25 \tfrac{1}{m}.$$

P2-characteristic is much slower than P1 and enters a disturbed region.
It is not essential for the critical behavior.

Micro-macro; numerical results

Gassmann-type relations - macroparameters in function of porosity:

$$\delta = \frac{K_V - K}{n\left(K_s - K_f\right)}, \quad K = \lambda_0^S + \frac{2}{3}\mu_0^S + \rho_0^F \kappa_0, \quad K_V = (1-n)K_s + nK_f,$$

$$\lambda^S + 2\mu^S = \frac{3(1-\nu)}{1+\nu}\left\{\frac{(K_s - K_d)^2}{\frac{K_s^2}{K_W} - K_d} + K_d\right\}, \quad \frac{1}{K_W} = \frac{1-n}{K_s} + \frac{n}{K_f},$$

$$\mu^S = \frac{3(1-2\nu)}{2(1+\nu)}\left\{\frac{(K_s - K_d)^2}{\frac{K_s^2}{K_W} - K_d} + K_d\right\},$$

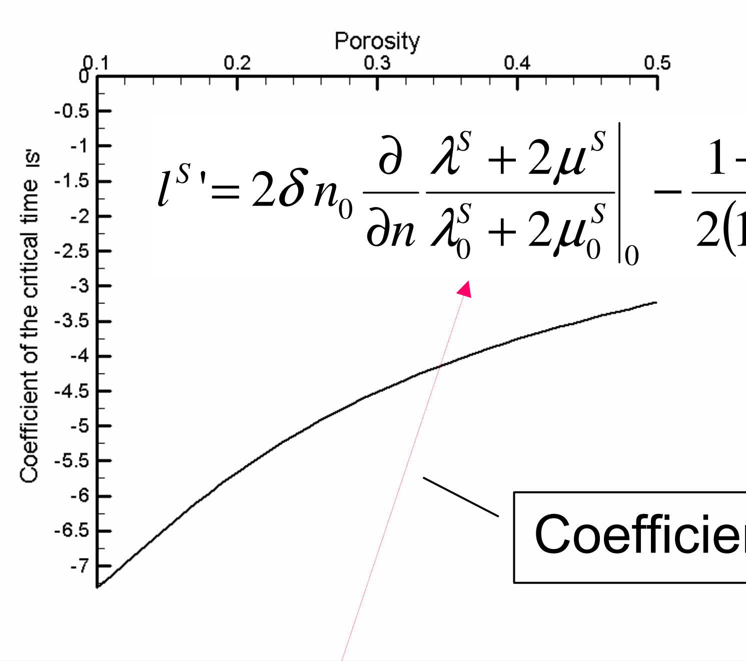
$$\rho_0^F \kappa = n^2 \frac{K_s^2}{\frac{K_s^2}{K_W} - K_d}, \qquad K_d = \frac{K_s}{1+50n}.$$

Geertsma empirical relation

Given:

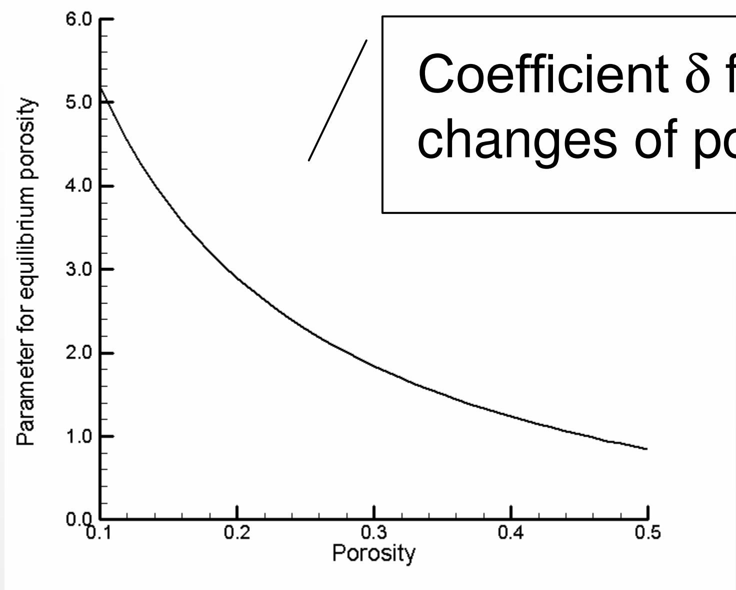
$$K_s, K_f, \nu$$

K. Wilmanski; *On microstructural tests for poroelastic materials and corresponding Gassmann-type relations*, *Gèotechnique*, **54**, 9, 593-603 (2004).

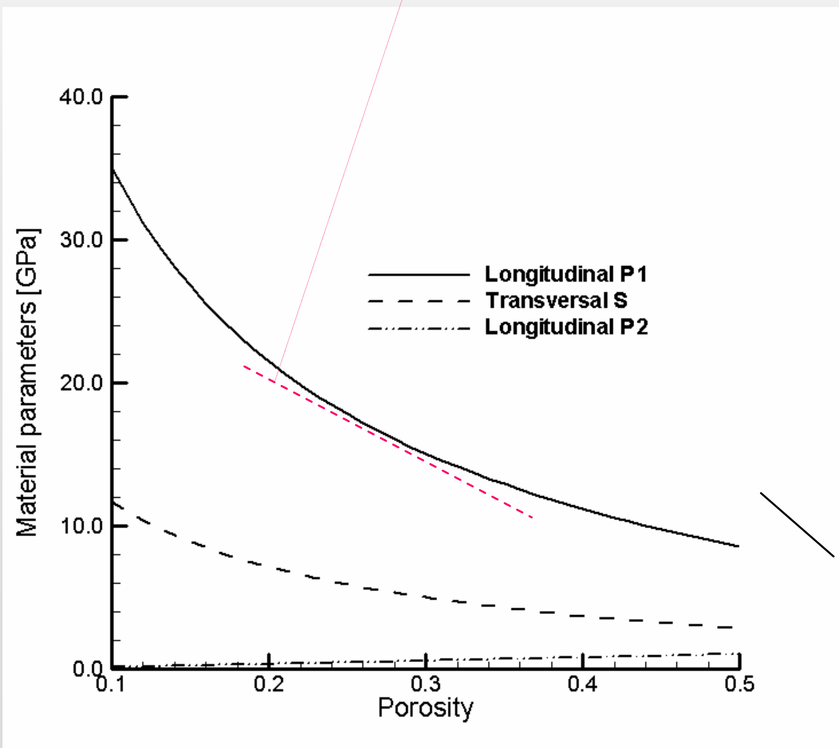


$$l^{S}{}' = 2\delta n_0 \frac{\partial}{\partial n} \frac{\lambda^S + 2\mu^S}{\lambda_0^S + 2\mu_0^S}\bigg|_0 - \frac{1+2\nu}{2(1-\nu)} < 0.$$

Coefficient $l^{S}{}'$



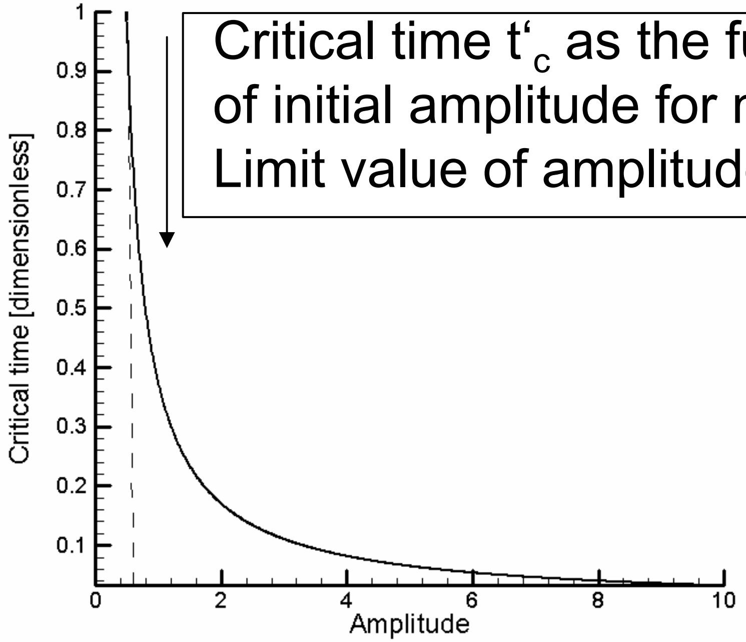
Coefficient δ for changes of porosity



Material parameters:
$\lambda^S + 2\mu^S, \quad \mu^S, \quad \rho_0^F \kappa$

Numerical data:

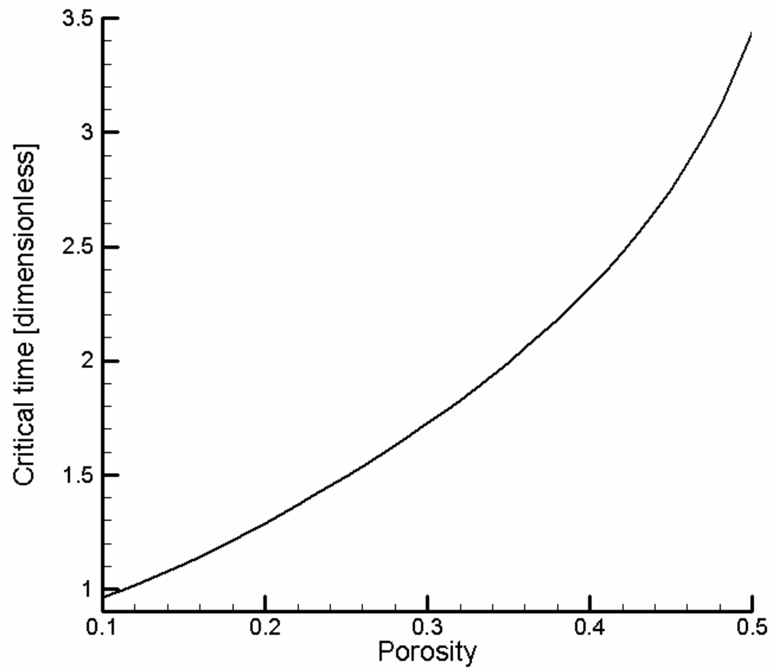
$$K_s = 48 GPa, \quad K_f = 2.25 GPa,$$
$$\nu = 0.25, \quad K_d = \frac{K_s}{1+50n}.$$



Critical time t'_c as the function of initial amplitude for n=0.25
Limit value of amplitude: 0.3133

Critical time t‘ as the function of porosity for the initial amplitude

$$\mathcal{A}_0 = \frac{\sqrt{2}}{2 - \frac{1}{2} l^{S\prime}(n = 0.55)}.$$



Critical distance: $x_c = \frac{2\rho_0^S c_{P1}}{\pi} t'_c$

For ρ^S_0=2500 kg/m^3, c_{P1}=2500 m/s,
π=10^6 kg/m^{3}s $\Rightarrow x = 12.5\, t'_c$ m

Quantitative agreement with results of Osinov within hypoplasticity for Karlsruhe sand

V. A. Osinov: *On the formation of discontinuities of wave fronts in a saturated granular body*, Cont. Mech. Termodyn., **10**, 253-268, 1998

Derivation of micro-macro relations for compressibilities

Geometrical compatibility

for partial mass densities in homogeneous microstructure

e.g.:

$$\rho^F = \frac{1}{V} \int_{REV(\mathbf{x})} \rho^{FR}(\mathbf{z},t) H(\mathbf{z},t)\, dV_{\mathbf{z}} \equiv n(\mathbf{x},t)\rho^{FR}(\mathbf{x},t),$$

$$n(\mathbf{x},t) := \frac{1}{V} \int_{REV(\mathbf{x})} H(\mathbf{z},t)\, dV_{\mathbf{z}}, \quad V := \text{volume } REV$$

homogeneity

where $H(\mathbf{z},t)$ is the characteristic function for the fluid component. Then

$$\rho^F = n\rho^{FR}, \quad \rho^F = \rho_0^F (1+\varepsilon)^{-1}, \quad \rho^{FR} = \rho_0^{FR}\left(1+\varepsilon^R\right)^{-1} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{n}{n_0} = \frac{1+\varepsilon^R}{1+\varepsilon}, \quad n_0 = \frac{\rho_0^F}{\rho_0^{FR}},$$

$$\rho^S = (1-n)\rho^{SR}, \quad \rho^S = \rho_0^S (1+e)^{-1}, \quad \rho^{SR} = \rho_0^{SR}\left(1+e^R\right)^{-1} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{1-n}{1-n_0} = \frac{1+e^R}{1+e}.$$

Linearity (small deformations) yields **geometrical compatibility** conditions:

$$e = e^R + \frac{n-n_0}{1-n_0}, \quad \varepsilon = \varepsilon^R - \frac{n-n_0}{n_0}. \qquad (1)$$

Changes of porosity

Constitutive relations:

- macro

$$p^S - p_0^S = -(\lambda^S + \tfrac{2}{3}\mu^S)e - Q\varepsilon + \mathrm{N}(n\text{-}n_0),$$

$$p^F - p_0^F = -Qe - \rho_0^F \kappa\varepsilon - \mathrm{N}(n\text{-}n_0),$$

$$\text{equilibrium}: \quad \Delta \mathrm{p} = (p^S - p_0^S) + (p^F - p_0^F) =$$

$$= -(\lambda^S + \tfrac{2}{3}\mu^S + Q)e - (\rho_0^F \kappa + Q)\varepsilon. \qquad (2)$$

- micro

$$p^{FR} - p_0^{FR} = -K_f \varepsilon^R, \quad p^{SR} - p_0^{SR} = -K_s e^R,$$

$$\text{equilibrium}: \quad \Delta \mathrm{p} = \mathrm{n}_0 (p^{FR} - p_0^{FR}) + (1 - n_0)(p^{SR} - p_0^{SR}). \qquad (3)$$

It follows

$$\boxed{\frac{n - n_0}{n_0} = \delta e + \gamma(e - \varepsilon)}, \quad \delta := \frac{K_V - K}{n_0 (K_s - K_f)}, \quad \gamma := \frac{\rho_0^F \kappa + Q - n_0 K_f}{n_0 (K_s - K_f)},$$

$$K_V := (1 - n_0) K_s + n_0 K_f, \quad K := \lambda^S + \tfrac{2}{3}\mu^S + \rho_0^F \kappa + 2Q.$$

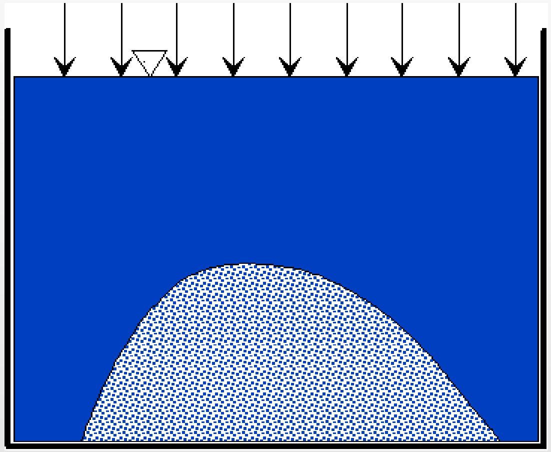
Gedankenexperiments for homogeneous microstructures

Unknown: $\left\{e, \zeta, n, e^R, \varepsilon^R, p-p_0, p_f-p_f^0\right\}$ **= 7;**

Equations: 2 geom., 2 equilib., 4 constit. = 8.

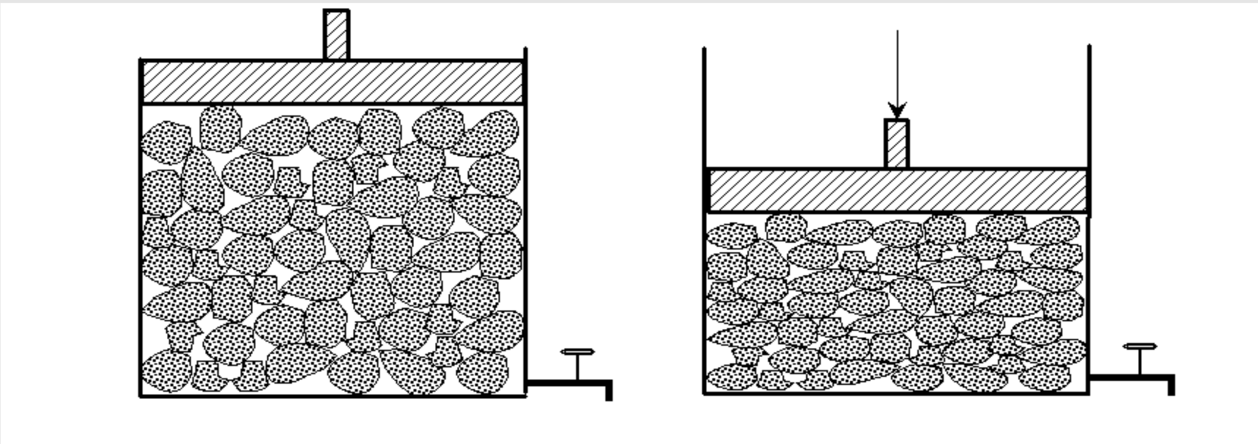
General equilibrium conditions:

$$\Delta p = (p^S - p_0^S) + (p^F - p_0^F) =$$
$$= (1-n_0)(p^{SR} - p_0^{SR}) + n_0(p^{FR} - p_0^{FR}).$$



Unjacketed test:

$$p_f - p_f^0 = \Delta p,$$



Jacketed drained

$$p_f - p_f^0 = 0,$$

and undrained tests:

$$\zeta = 0, \quad \text{i.e.} \quad \mathrm{e} = \varepsilon.$$

Solutions of field equations and geometrical compatibility

jacketed undrained

$$e = -\frac{\Delta p}{K}, \quad \zeta = 0, \quad \frac{n - n_0}{n_0} = -\frac{C - K_f}{K(K_f - \mathrm{N})}\Delta p,$$

$$C > K_f \quad \Rightarrow \quad \mathrm{N} < K_f .$$

$$p_f - p_f^0 = \left(\frac{C}{K} + \mathrm{N}\frac{C - K_f}{K(K_f - \mathrm{N})}\right)\Delta p,$$

$$K = K_V - n_0 (K_s - K_f)\frac{C - K_f}{K_f - \mathrm{N}}, \qquad (1)$$

$$K_V := (1 - n_0) K_s + n_0 K_f .$$

jacketed drained

$$e = -\frac{\Delta p}{K_b} - \frac{\mathrm{N}C}{K_b M}\frac{\Delta p}{K_n},$$

$$\zeta = -\frac{C}{K_b M}\left(1 + \frac{K\mathrm{N}}{CK_n}\right)\Delta p,$$

$$\frac{n - n_0}{n_0} = -K_n \Delta p,$$

$$K_b := K - \frac{C^2}{M}, \qquad (4)$$

$$K_n := K_s \frac{(1 - n_0)\frac{\mathrm{N}C}{K_b M} - n_0}{1 - (1 - n_0)\frac{K_s}{K_b}},$$

$$\frac{K_s}{K_b}\left(n_0 - \frac{C}{M}\right) - \frac{K_s}{K_n}\left(n_0 - \frac{\mathrm{N}(K + C)}{K_b M}\right) = 0, \qquad (2)$$

unjacketed

$$e = -\frac{1 - \frac{C}{K_W}}{1 - \frac{C}{K}}\frac{\Delta p}{K}, \quad \zeta = -\frac{1 - \frac{K}{K_W}}{1 - \frac{C}{K}}\frac{\Delta p}{K},$$

$$\frac{n - n_0}{1 - n_0} = \left(\frac{K}{K_s} - -\frac{1 - \frac{C}{K_W}}{1 - \frac{C}{K}}\right)\frac{\Delta p}{K},$$

$$K = \frac{C - M + \frac{MK_b}{K_W}}{1 - \frac{C}{K}} - \mathrm{N}\frac{1 - n_0}{n_0}\left(\frac{K}{K_s} - \frac{1 - \frac{C}{K_W}}{1 - \frac{C}{K}}\right), \qquad (3)$$

$$\frac{1}{K_W} := \frac{1 - n_0}{K_s} + \frac{n_0}{K_f}.$$

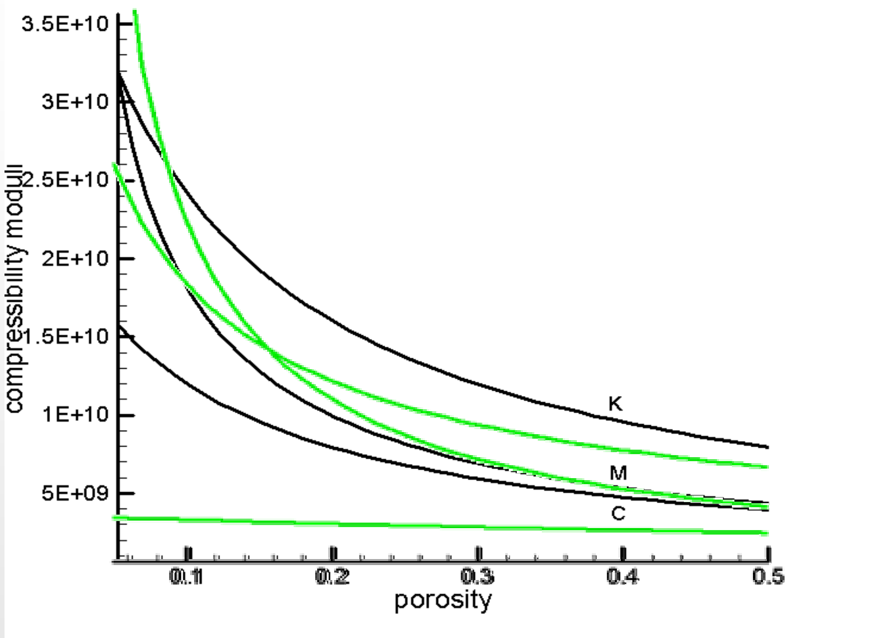
Full set of equations for *K, C, M, N*:

$$K = K_V - n_0 (K_s - K_f) \frac{C - K_f}{K_f - \mathrm{N}}, \quad K_V := (1 - n_0) K_s + n_0 K_f,$$

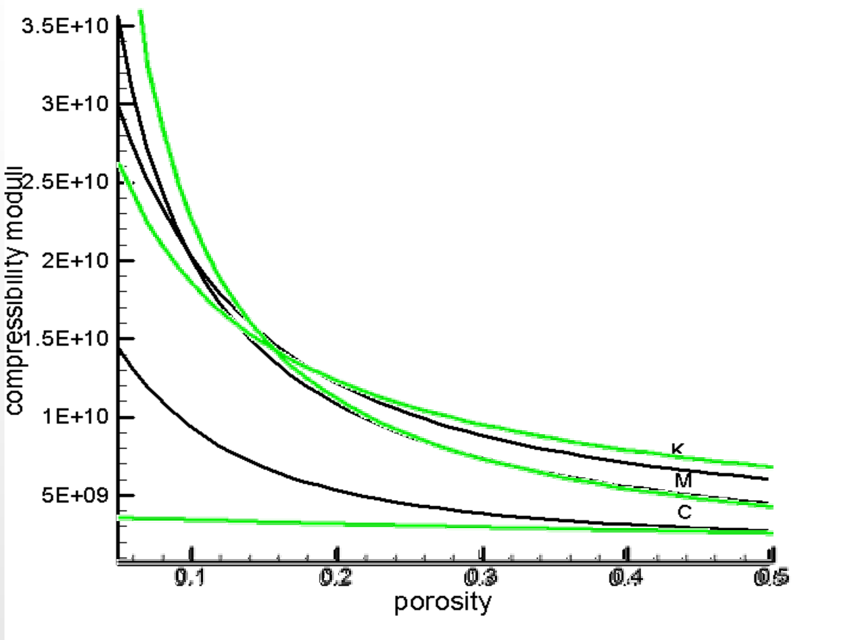
$$\frac{K_s}{K_b}\left(n_0 - \frac{C}{M}\right) - \frac{K_s}{K_n}\left(n_0 - \frac{\mathrm{N}(K + C)}{K_b M}\right) = 0, \quad K_b := K - \frac{C^2}{M},$$

$$K = \frac{C - M + \frac{M K_b}{K_W}}{1 - \frac{C}{K}} - \mathrm{N}\,\frac{1\text{-}n_0}{n_0}\left(\frac{K}{K_s} - \frac{1 - \frac{C}{K_W}}{1 - \frac{C}{K}}\right), \quad \frac{1}{K_W} := \frac{1 - n_0}{K_s} + \frac{n_0}{K_f},$$

$$K_d = K_b \left\{1 + \frac{\mathrm{N} C}{K_\mathrm{b} M} \frac{1}{K_n}\right\}^{-1}, \quad K_n := K_s \frac{(1 - n_0)\frac{\mathrm{N} C}{K_\mathrm{b} M} - n_0}{1 - (1 - n_0)\frac{K_s}{K_b}}.$$



Material parameters *K, C, M* in the zeroth approximation (Gassmann)



Material parameters *K, C, M* in the first approximation (Gassmann)

Comparison with the full solution (green)

Concluding remarks

Dependence of material parameters on porosity yields the existence of the threshold if the initial ampitude produces tension.

The size of the critical time and the intensity of tensile amplitude depends on the slope of the diagram of material parameters in function of porosity.