The Earth

The Water

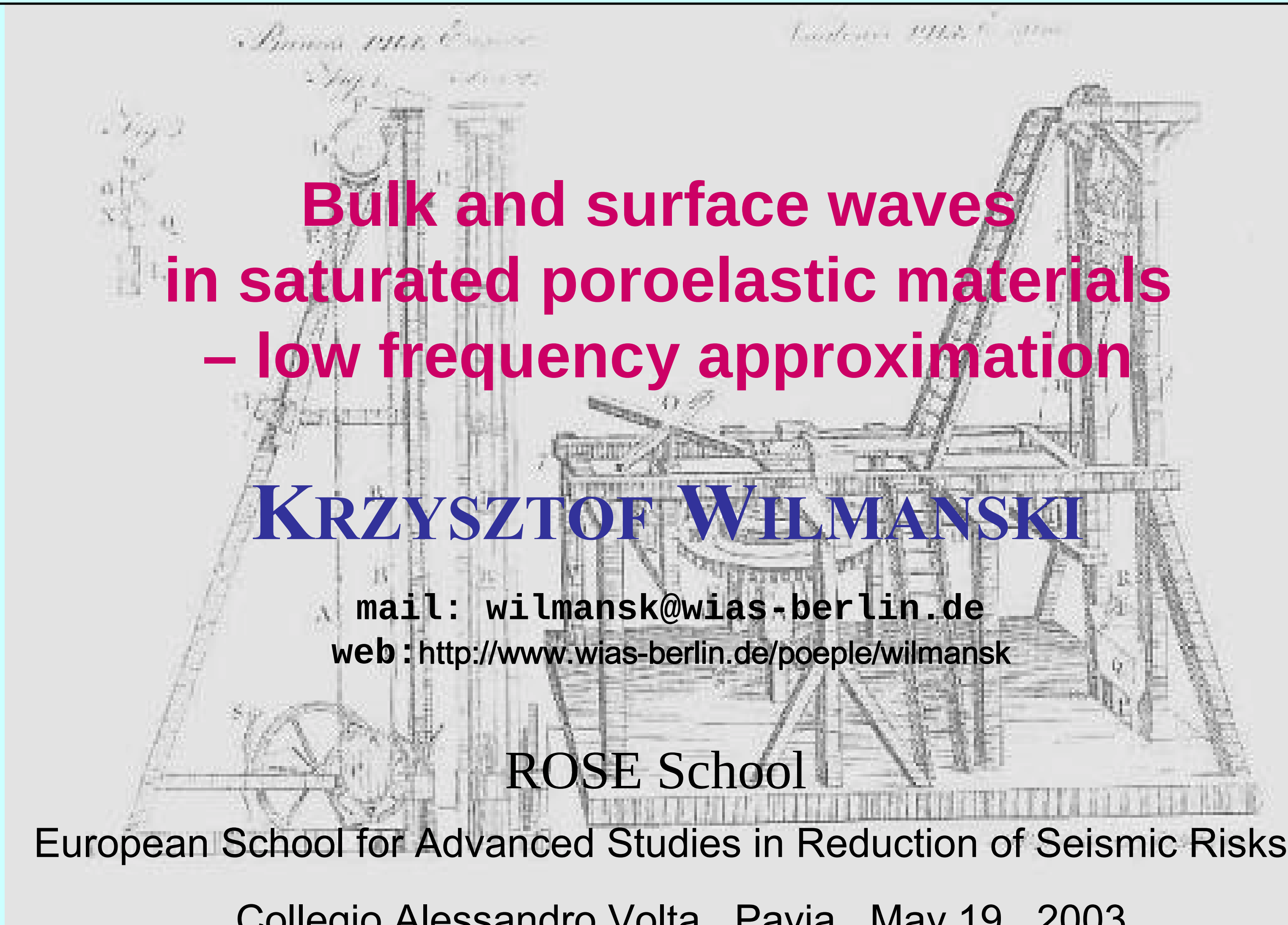
Giuseppe Arcimboldo
(1570)

Leibniz Gemeinschaft

WIAS

Weierstrass Institute for Applied Analysis and Stochastics

in Forschungsverbund Berlin e.V., Mohrenstrasse 39, D - 10117 Berlin, Germany



Bulk and surface waves in saturated poroelastic materials – low frequency approximation

KRZYSZTOF WILMANSKI

mail: wilmansk@wias-berlin.de

web: http://www.wias-berlin.de/poeple/wilmansk

ROSE School

European School for Advanced Studies in Reduction of Seismic Risks

Collegio Alessandro Volta, Pavia, May 19, 2003

Contents:

1) Preliminary remarks on waves

Waves as a tool for inquiry of structure of complex materials!

Bulk and surface waves

Dr. **Dan Russel**, Kettering University Applied Physics

http://www.kettering.edu/~drussell/Demos/reflect/reflect.html

Longitudinal wave

Transversal wave

Mexican wave

Water wave

Rayleigh wave

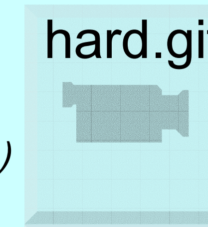
Influence of boundaries and interfaces

Influence of boundary condition on transmission and reflexion - impedance

Dr. **Dan Russel**, Kettering University, Applied Physics

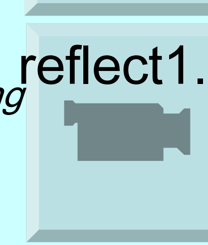
http://www.kettering.edu/~drussell/Demos/reflect/reflect.html

at a fixed (hard) boundary, the displacement remains zero (Dirichlet conditions) and the reflected wave changes its polarity (undergoes a 180^0 phase change)

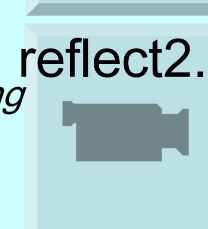


at a free (soft) boundary, the restoring force is zero (Neumann conditions) and the reflected wave has the same polarity (no phase change) as the incident wave

Permeable boundary: the incident wave is travelling from a region of low impedance towards a high impedance region



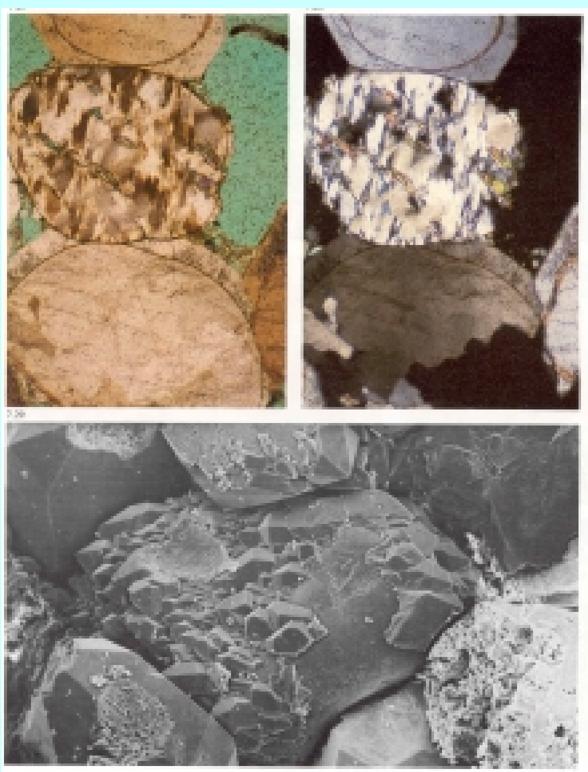
Permeable boundary: the incident wave is travelling from a high impedance region towards a low impedance region



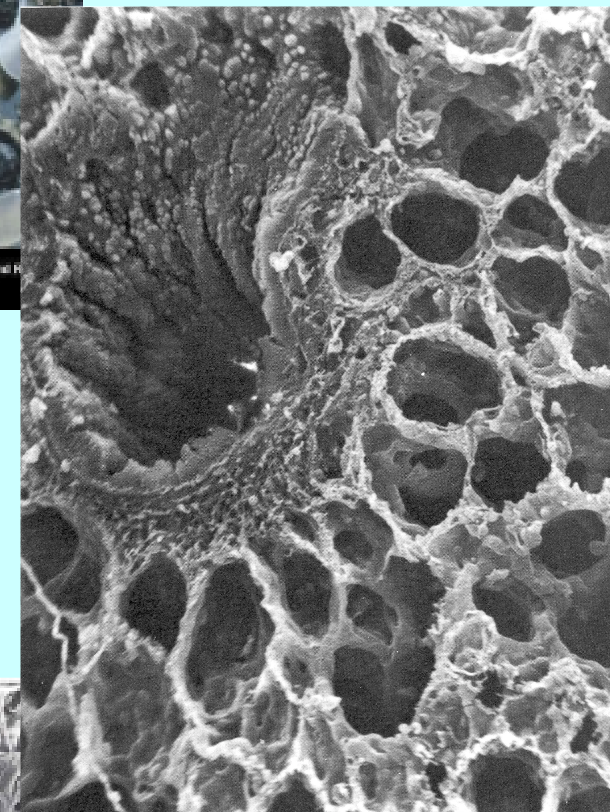
Some examples of complex media

Sand

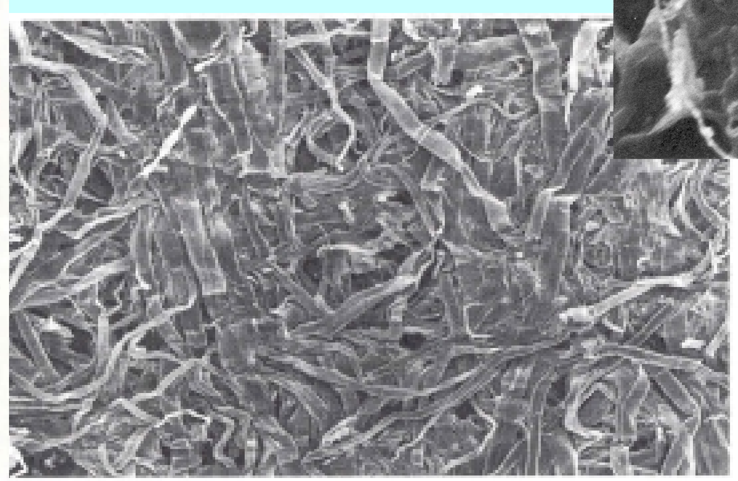
Sandstone in diagenesis



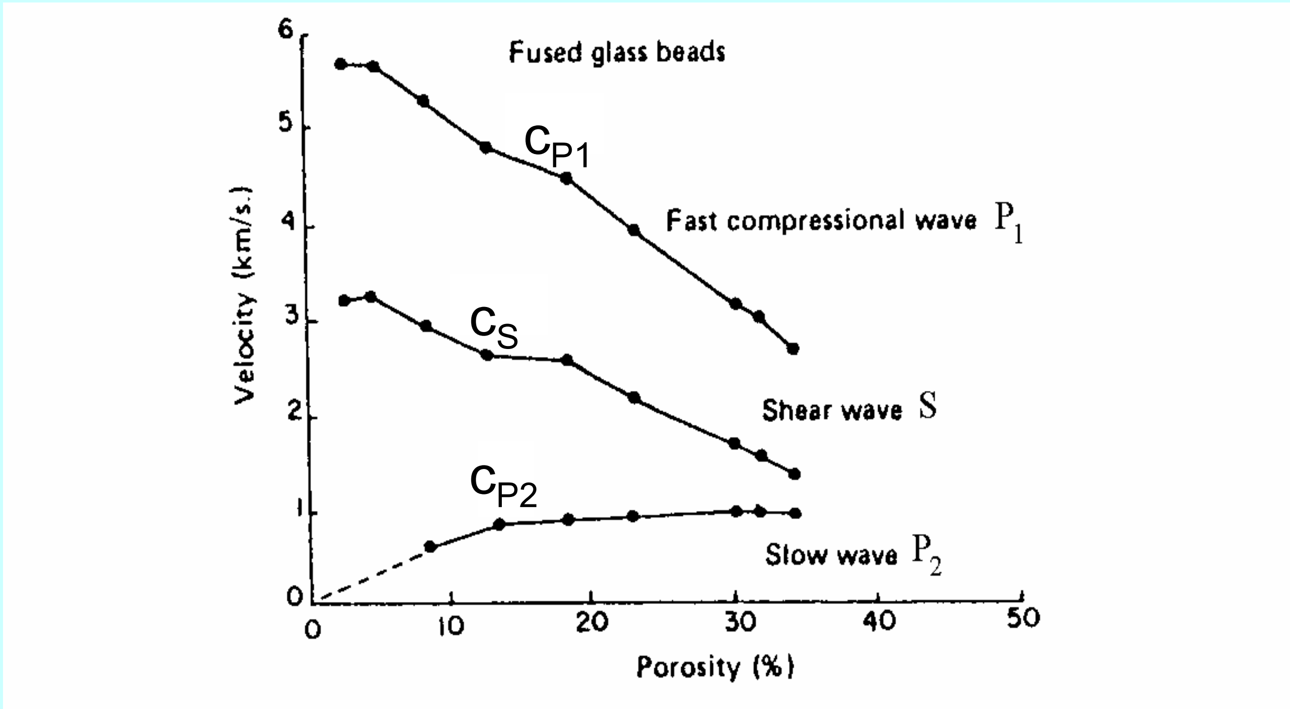
Lungs



Toilet paper



2) Experimental measurements of bulk waves in porous materials



Velocities of bulk waves in an artificial porous material (sintered glass);

T. J. PLONA, D. L. JOHNSON, Experimental Study of the Two Bulk Compres-Sional Modes in Water-saturated Porous Structures, Ultrasonic Symp. IEEE, 868-872, 1980.

3) Modeling of porous materials –
- linear poroelastic two-component medium

Governing equations – linear poroelastic model

Balance equations

$$\frac{\partial \rho^S}{\partial t} + \rho_0^S \, div \, \mathbf{v}^S = 0, \quad \frac{\partial \rho^F}{\partial t} + \rho_0^F \, div \, \mathbf{v}^F = 0,$$

$$\rho_0^S \frac{\partial \mathbf{v}^S}{\partial t} = div \, \mathbf{T}^S + \pi (\mathbf{v}^F - \mathbf{v}^S) + \rho^S \mathbf{b}^S,$$

$$\rho_0^F \frac{\partial \mathbf{v}^F}{\partial t} = div \, \mathbf{T}^F - \pi (\mathbf{v}^F - \mathbf{v}^S) + \rho^F \mathbf{b}^F,$$

$$\frac{\partial (n - n_E)}{\partial t} + \Phi \, div (\mathbf{v}^F - \mathbf{v}^S) = -\frac{n - n_E}{\tau}.$$

Constitutive relations: $\tau \longrightarrow \infty$

Biot

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S e \mathbf{1} + 2\mu^S \mathbf{e}^S + Q \varepsilon \mathbf{1} - \mathrm{N}(n - n_0)\mathbf{1}, \quad e := tr \, \mathbf{e}^S, \quad \frac{\partial \mathbf{e}^S}{\partial t} = sym \, grad \, \mathbf{v}^S,$$

$$\mathbf{T}^F = -p^F \mathbf{1} + \mathrm{N}(n - n_0)\mathbf{1}, \quad p^F = p_0^F - Qe - \rho_0^F \kappa \varepsilon, \quad \varepsilon := \frac{\rho_0^F - \rho^F}{\rho_0^F}, \quad n_E = n_0 (1 + \delta e).$$

Solution of the porosity balance:

$$\frac{n-n_0}{n_0}(t,\mathbf{x}) = \delta\, e(t,\mathbf{x}) + \gamma\left(e(t,\mathbf{x}) - \varepsilon(t,\mathbf{x})\right) -$$

$$-\frac{\gamma}{\tau}\int_0^t \left(e(t-s,\mathbf{x}) - \varepsilon(t-s,\mathbf{x})\right) e^{-\frac{s}{\tau}}\, ds, \quad \gamma := \frac{\Phi}{n_0}.$$

$= 0 \quad \text{for} \quad \tau \to \infty$

Effective stress – - strain relations:

$$\mathbf{T}^S = \mathbf{T}_0^S + \left[\lambda^S - n_0 \mathrm{N}(\delta+\gamma)\right] e\mathbf{1} + 2\mu^S \mathbf{e}^S + (Q + n_0 \mathrm{N}\gamma)\varepsilon\mathbf{1},$$

$$p^F = p_0^F - [Q + n_0\mathrm{N}(\delta+\gamma)]e - (\rho_0^F\kappa - n_0\mathrm{N}\gamma)\varepsilon.$$

Approximation justified by generalized Gassmann relations: $\mathrm{N} \approx 0.$

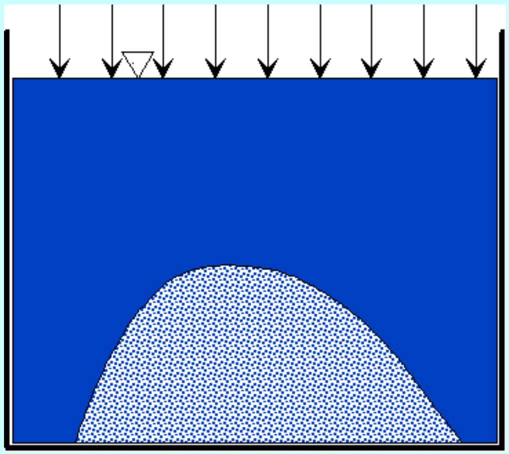
Field equations:

$$\rho_0^S \frac{\partial \mathbf{v}^S}{\partial t} = \lambda^S \operatorname{grad} e + 2\mu^S \operatorname{div} \mathbf{e}^S + Q \operatorname{grad}\varepsilon + \pi(\mathbf{v}^F - \mathbf{v}^S),$$

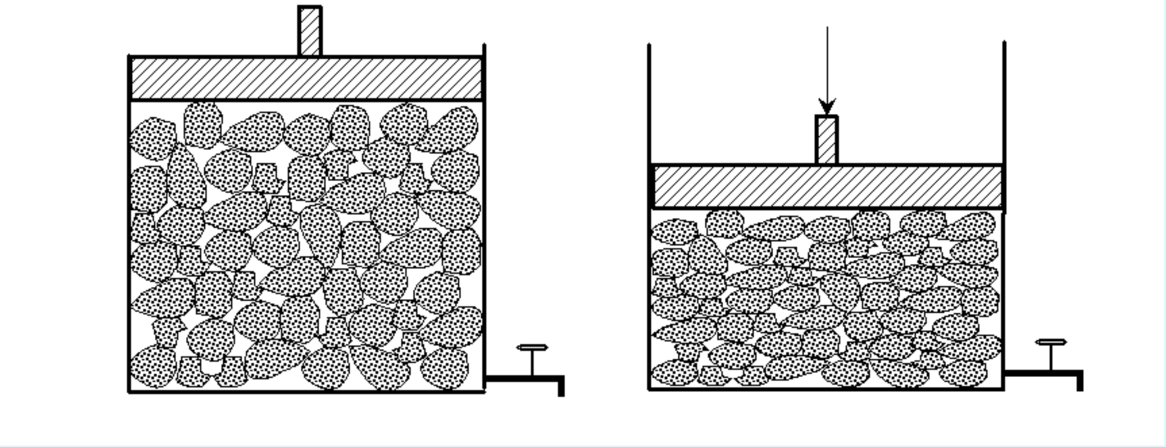
$$\rho_0^F \frac{\partial \mathbf{v}^F}{\partial t} = Q \operatorname{grad} e + \rho_0^F \kappa \operatorname{grad}\varepsilon - \pi(\mathbf{v}^F - \mathbf{v}^S),$$

$$\frac{\partial \mathbf{e}^S}{\partial t} = \operatorname{sym}\operatorname{grad} \mathbf{v}^S, \quad \frac{\partial \varepsilon}{\partial t} = \operatorname{div} \mathbf{v}^F, \quad \frac{\partial}{\partial t}\left[n - n_0\delta e + \Phi(e-\varepsilon)\right] = -\frac{n - n_0\delta e}{\tau}.$$

Micro-macrotransitions: Gedankenexperiments for homogeneous microstructures



Unjacketed test



Jacketed drained and undrained tests

Gassmann relations – coupling in Biot's model

Macroscopic material parameters of Biot's model

$$K := \lambda^S + \tfrac{2}{3}\mu^S + \rho_0^F \kappa + 2Q, \quad M := \frac{\rho_0^F \kappa}{n_0^2}, \quad C := \frac{1}{n_0}\left(Q + \rho_0^F \kappa\right).$$

Classical Gassmann relations

$$K = \frac{(K_s - K_b)^2}{D}, \quad C = \frac{K_s(K_s - K_b)}{D}, \quad M = \frac{K_s^2}{D},$$

$$D := \frac{K_s^2}{K_W} - K_b, \quad \frac{1}{K_W} := \frac{1-n_0}{K_s} + \frac{n_0}{K_f},$$

where

K_s - bulk real compressibility modulus of skeleton,

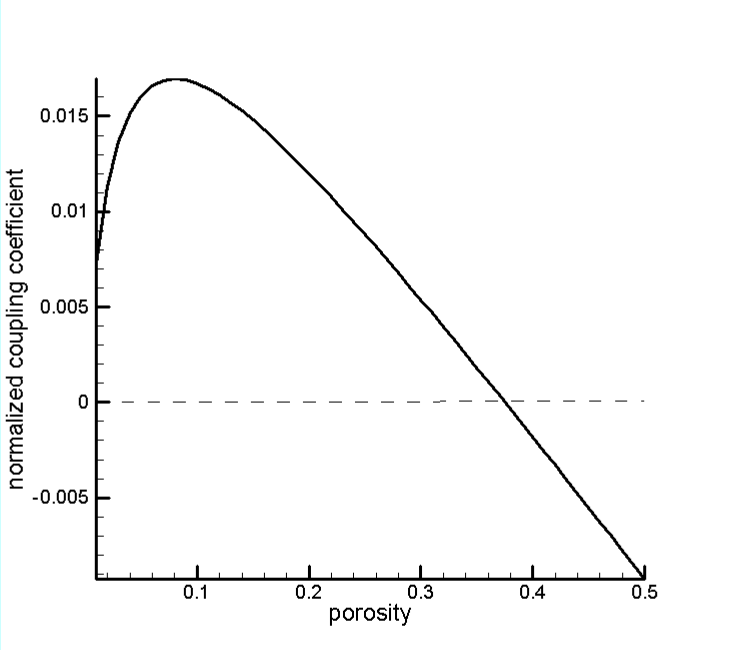
K_f - real compressibility modulus of fluid,

K_b - drained compressibility modulus.

Porosity coefficients

$$\delta = \frac{(1-n_0)K_s + n_0 K_f - K}{n_0(K_s - K_f)}, \quad \gamma = \frac{C - K_f}{K_s - K_f}.$$

Numerical example

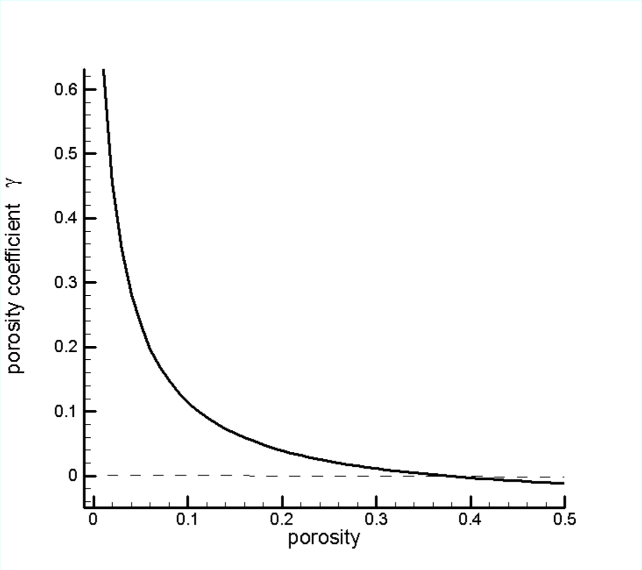


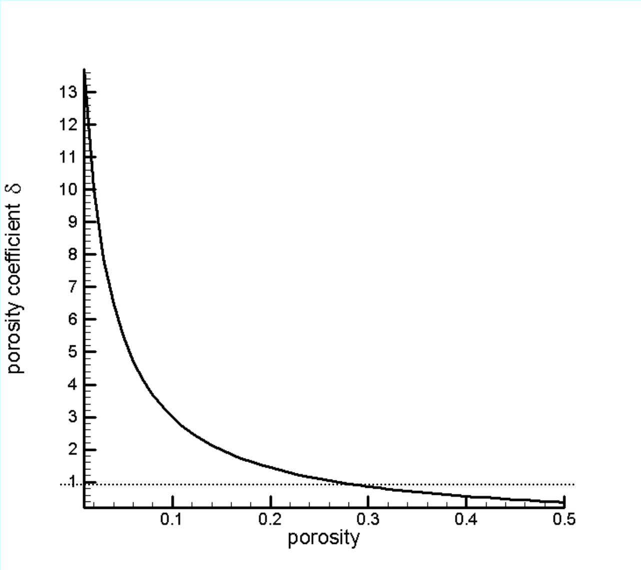
Normalized coupling coefficient $\frac{Q}{K}$

Data:

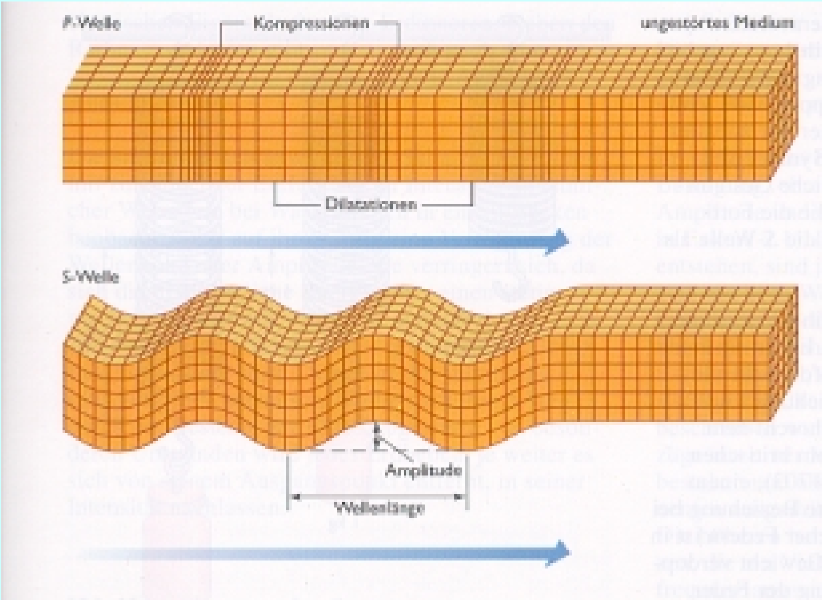
$$K_s = 48\times10^{10}\,Pa,$$

$$K_f = 2.25\times10^{10}\,Pa$$





Coefficients γ and δ in the relation for porosity



4) Sound waves in two-component poroelastic media

Monochromatic bulk waves – infinite media

$$\mathbf{v}^S = \mathbf{V}^S \mathsf{E}, \quad \mathbf{v}^F = \mathbf{V}^F \mathsf{E}, \quad \mathbf{e}^S = \mathbf{E}^S \mathsf{E}, \quad \varepsilon = E\mathsf{E},$$

$$\mathsf{E} := \exp i(\mathbf{x}\cdot\mathbf{k} - \omega t).$$

$\{\mathbf{V}^S, \mathbf{V}^F, \mathbf{E}^S, E\}$ - constant amplitude,

k – wave vector, ω - frequency.

Compatibility with field equations:

$$\begin{pmatrix} \rho_0^S\omega^2\mathbf{1} - \lambda^S\mathbf{k}\otimes\mathbf{k} - \mu^S(k^2\mathbf{1} + \mathbf{k}\otimes\mathbf{k}) + i\pi\omega\mathbf{1} & -[Q\mathbf{k}\otimes\mathbf{k} + i\pi\omega\mathbf{1}] \\ -[Q\mathbf{k}\otimes\mathbf{k} + i\pi\omega\mathbf{1}] & \rho_0^F\omega^2\mathbf{1} - \rho_0^F\kappa\mathbf{k}\otimes\mathbf{k} + i\pi\omega\mathbf{1} \end{pmatrix}\begin{pmatrix} \mathbf{V}^S \\ \mathbf{V}^F \end{pmatrix} = 0.$$

i.e. eigenvalue problem!

1) Boundary value problems in far field approximation:
- given real frequency ω (Fourier time expansion in monochromatic waves);

2) Initial value problem (e.g. impact): - given real wave vector *k* (Fourier integrals with respect to space variable)

1) Given frequency ω:

Separation of transversal and longitudinal waves:

S) $\mathbf{k}_\perp \cdot \mathbf{k} = 0 \quad \Rightarrow$

$$\begin{pmatrix} \rho_0^S \omega^2 - \mu^S k^2 + i\pi\omega & -i\pi\omega \\ -i\pi\omega & \rho_0^F \omega^2 + i\pi\omega \end{pmatrix} \begin{pmatrix} \mathbf{V}_\perp^S \\ \mathbf{V}_\perp^F \end{pmatrix} = 0, \quad \mathbf{V}_\perp^S := \mathbf{V}^S \cdot \mathbf{k}_\perp, \quad \mathbf{V}_\perp^F := \mathbf{V}^F \cdot \mathbf{k}_\perp.$$

i.e.

$$\left(\rho_0^S - \mu^S \frac{k^2}{\omega^2} \right) \rho_0^F + \frac{i\pi}{\omega} \left(\rho_0^S + \rho_0^F - \mu^S \frac{k^2}{\omega^2} \right) = 0.$$

a) Low frequency approximation $\omega \rightarrow 0:$

$$\left(\frac{\omega}{k} \right)^2 = \frac{\mu^S}{\rho_0^S + \rho_0^F}.$$

b) Propagation of the front - high frequency approximation $\omega \rightarrow \infty:$

$$\left(\frac{\omega}{k} \right)^2 = \frac{\mu^S}{\rho_0^S}.$$

L) $$\begin{pmatrix} \rho_0^S\omega^2 - \left(\lambda^S + 2\mu^S\right)k^2 + i\pi\omega & -i\pi\omega - Qk^2 \\ -i\pi\omega - qk^2 & \rho_0^F\omega^2 - \rho_0^F\kappa k^2 + i\pi\omega \end{pmatrix} \begin{pmatrix} \mathbf{V}^S \cdot \mathbf{k} \\ \mathbf{V}^F \cdot \mathbf{k} \end{pmatrix} = 0.$$

i.e.

$$\left(\rho_0^S - \left(\lambda^S + 2\mu^S\right)\frac{k^2}{\omega^2}\right)\left(\rho_0^F - \rho_0^F\kappa\frac{k^2}{\omega^2}\right) - Q^2\left(\frac{k^2}{\omega^2}\right)^2 +$$
$$+\frac{i\pi}{\omega}\left(\rho_0^S - \left(\lambda^S + 2\mu^S\right)\frac{k^2}{\omega^2} + \rho_0^F - \rho_0^F\kappa\frac{k^2}{\omega^2} - 2Q\frac{k^2}{\omega^2}\right) = 0.$$

a) Low frequency approximation $\omega \to 0:$

$$\left(\frac{\omega}{k}\right)^2 = \frac{\lambda^S + 2\mu^S + \rho_0^F\kappa + 2Q}{\rho_0^S + \rho_0^F}$$

- P1-wave; P2-wave does not propagate

b) Propagation of the front - high frequency approximation* $\omega \to \infty:$

$$Q \approx 0 \quad \Rightarrow \quad \left(\frac{\omega}{k}\right)^2 = \begin{cases} c_{P1}^2 := \dfrac{\lambda^S + 2\mu^S}{\rho_0^S}, \\ c_{P2}^2 := \kappa, \end{cases}$$

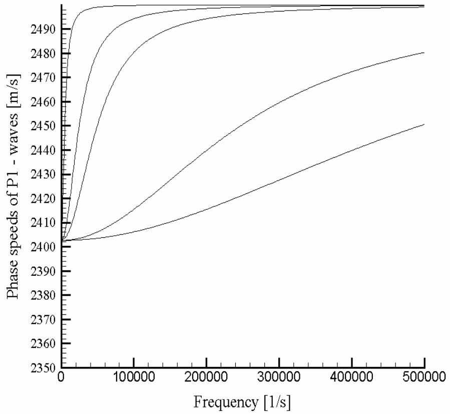
- P1-wave,

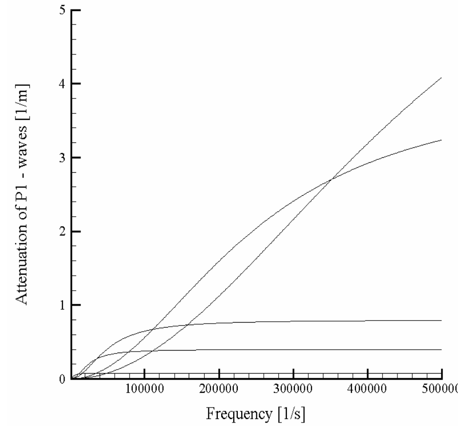
- P2-wave.

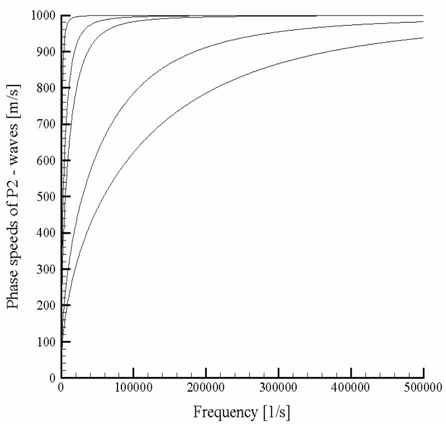
***)** $\omega \to \infty$:

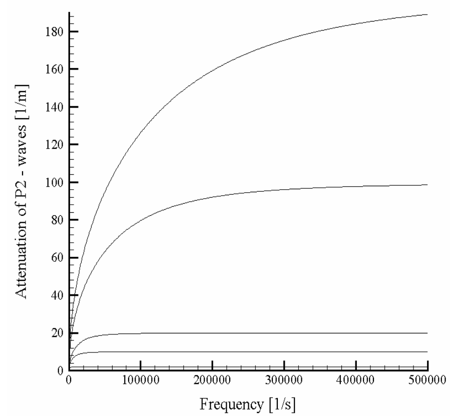
$$\frac{k^2}{\omega^2} = \frac{1}{2((\lambda^S + 2\mu^S)\rho_0^F \kappa - Q^2)} \left\{ \rho_0^F \left(\lambda^S + 2\mu^S \right) + \rho_0^S \rho_0^F \kappa \pm \right.$$

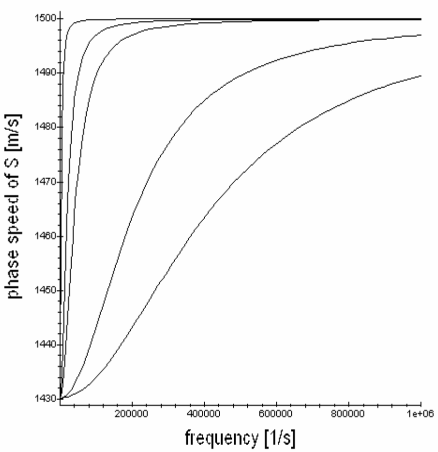
$$\left. \pm \sqrt{(\rho_0^F (\lambda^S + 2\mu^S) - \rho_0^S \rho_0^F \kappa)^2 + 4\rho_0^S \rho_0^F Q^2} \right\}.$$

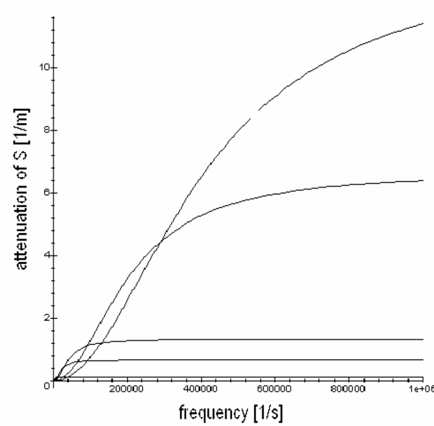








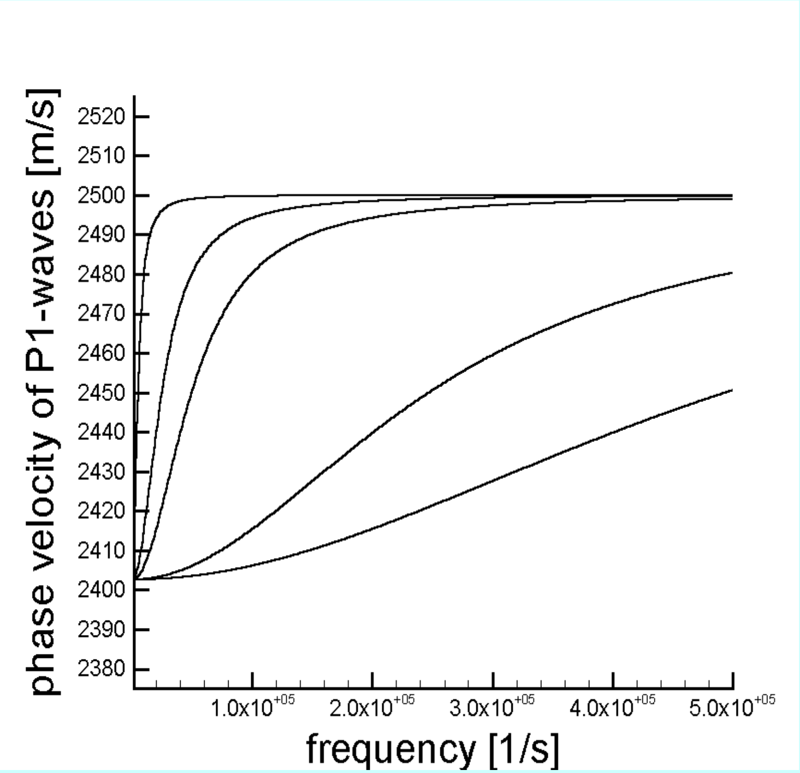


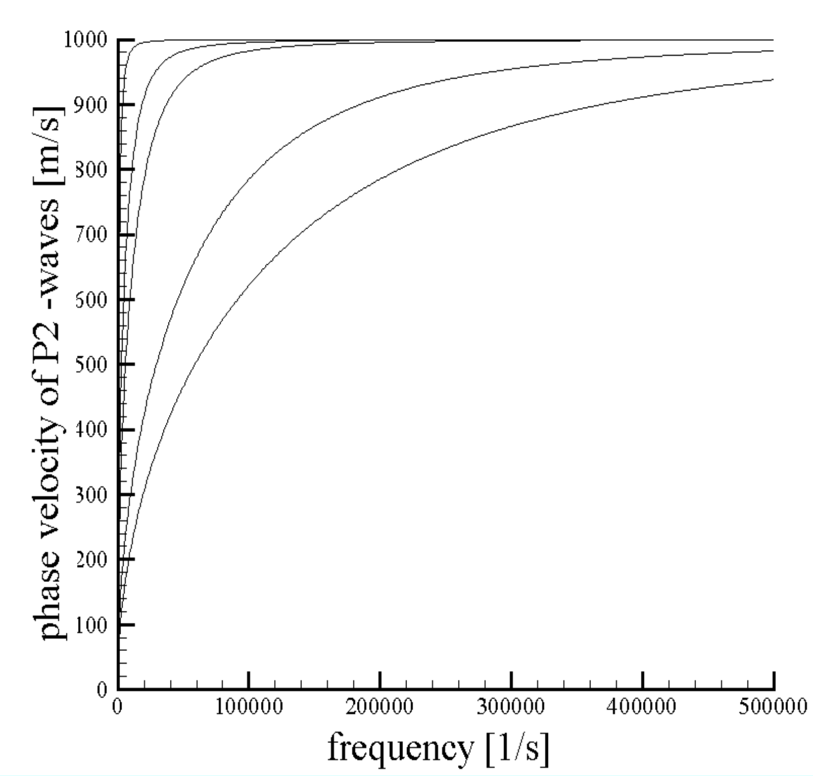


Phase speeds and attenuations

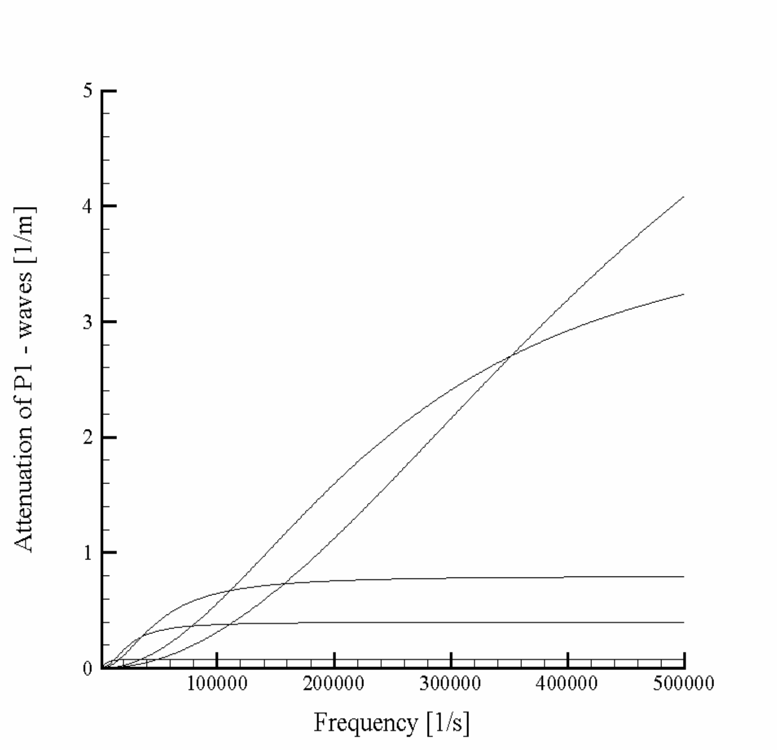
of P1, P2 , and S– waves
as functions of the frequency ω
for c_{P1} =2500 m/s, c_{P2} =1000 m/s,
c_S =1500 m/s,
ρ_0^S =2500 kg/m^3, ρ_0^F =250 kg/m^3,
permeabilities π =10^6, 5×10^6, 10^7,
5×10^7, 10^8 kg/m^{3}s,
Q=0.

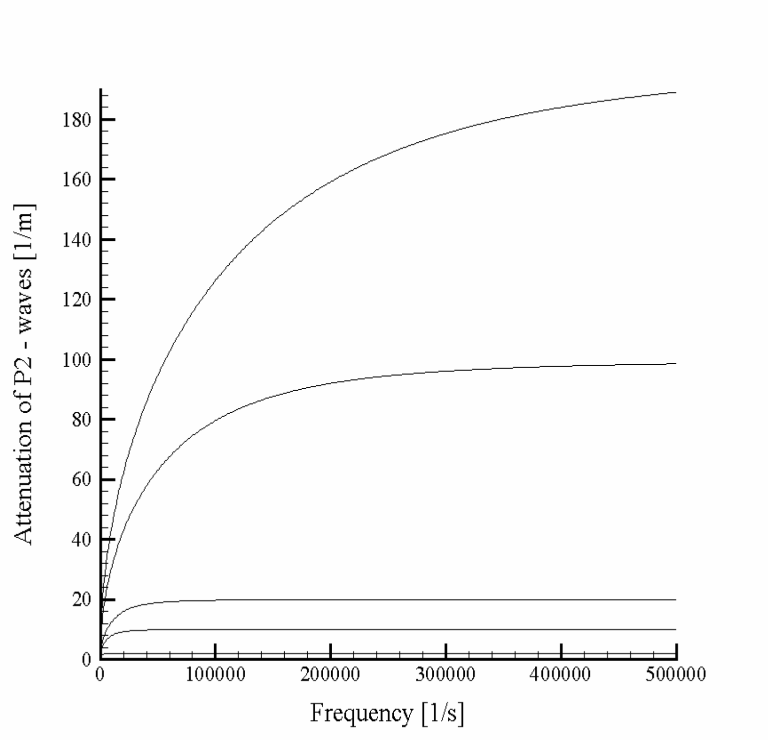
Phase velocities of monochromatic P1- and P2 – waves



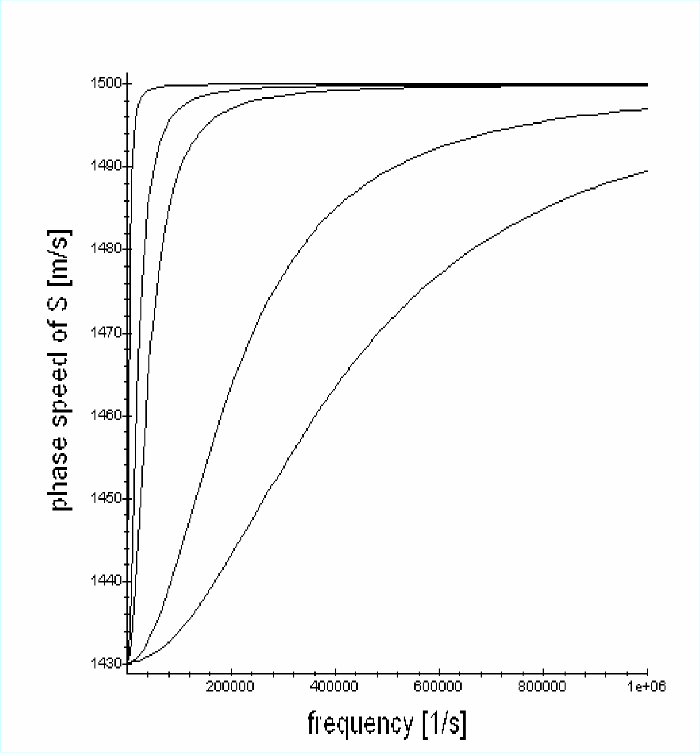


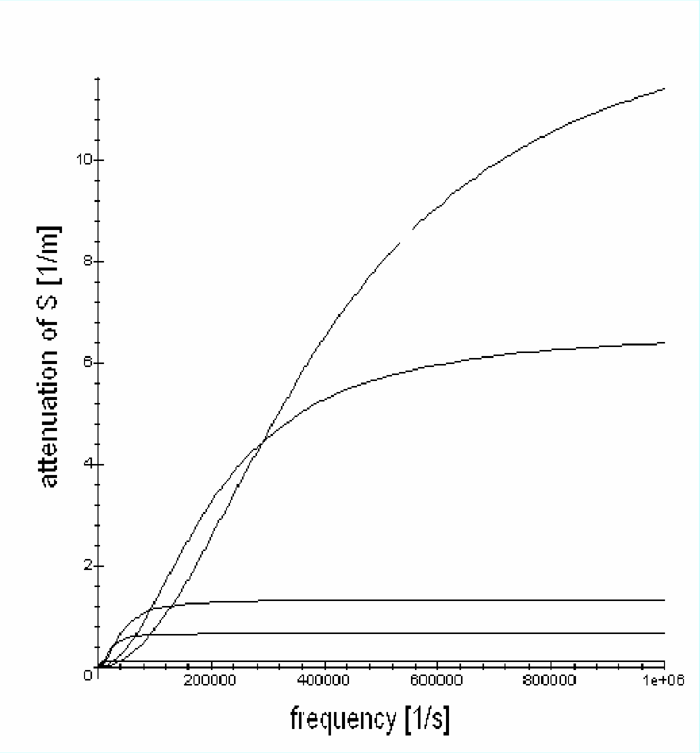
Attenuation of monochromatic P1 – and P2 - waves





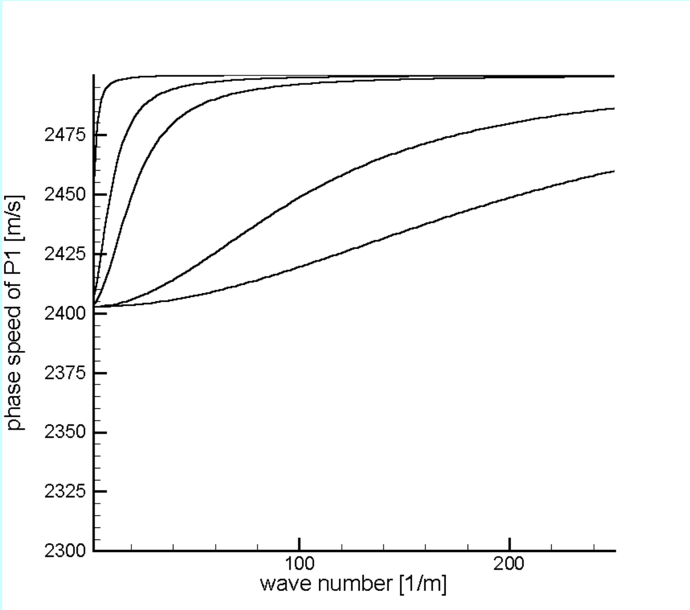
Phase velocity and attenuation of monochromatic S -waves



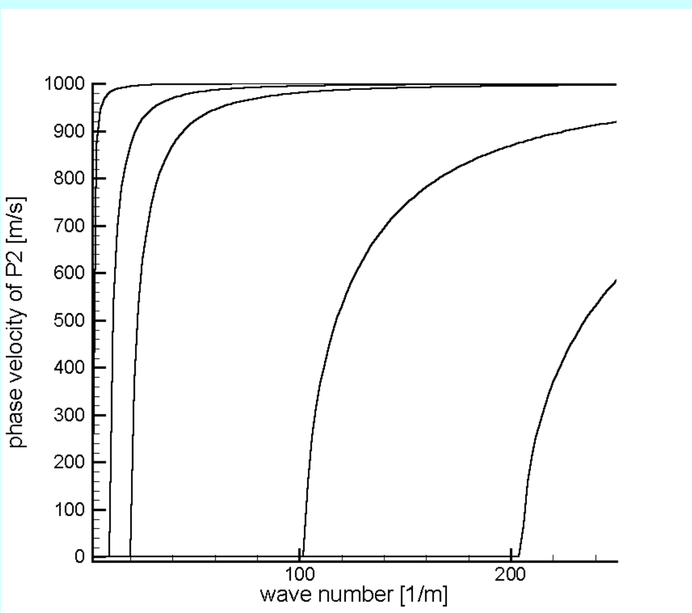


2) Given wave number *k*:

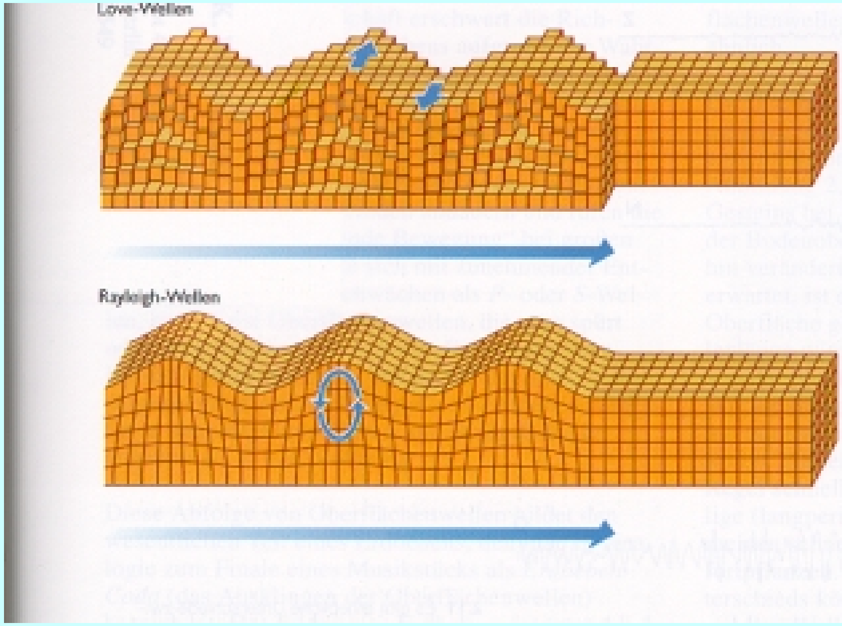
Numerical data:
c_{P1} =2500 m/s, c_{P2} =1000 m/s,
c_S =1500 m/s,
$\rho_0{}^S$ =2500 kg/m^3, $\rho_0{}^F$ =250 kg/m^3,
permeabilities π =10^6, 5×10^6, 10^7,
5×10^7, 10^8 kg/m^{3}s,
Q=0.



Phase speed of P1-waves

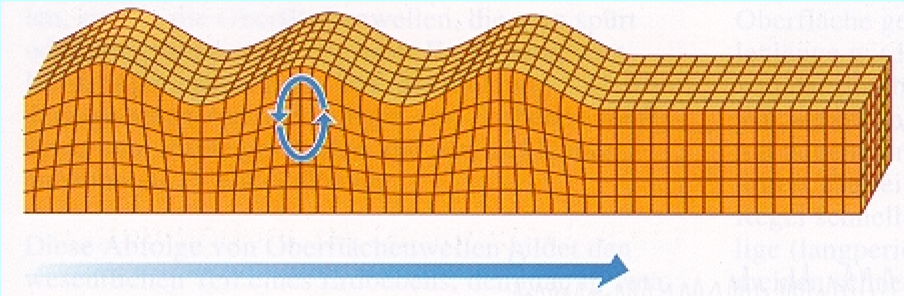


Phase speed of P2-waves –
- existence of a critical wave length



5) Surface waves in two-component poroelastic media

Rayleigh and Love waves in homogeneous elastic media



Rayleigh wave

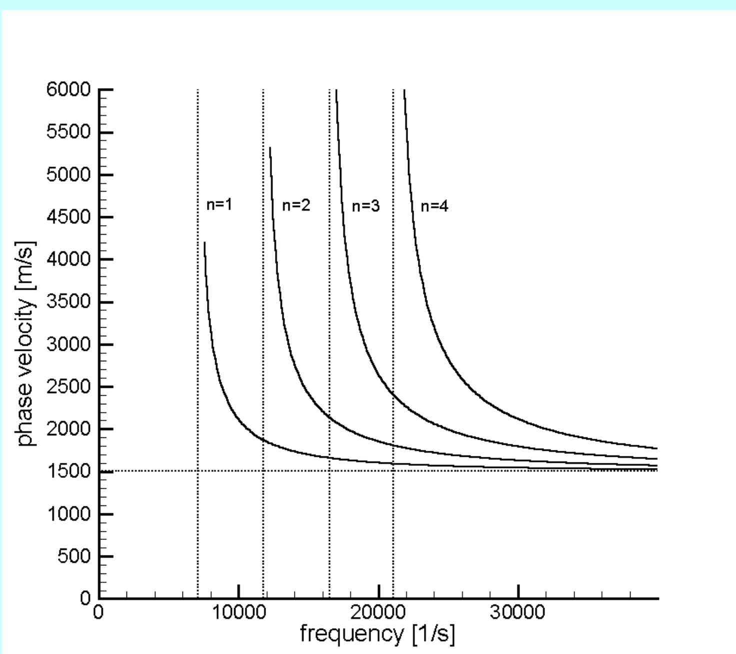
$$P_R := \left(2 - \frac{c_R^2}{c_T^2}\right)^2 - 4\sqrt{1 - \frac{c_R^2}{c_T^2}}\sqrt{1 - \frac{c_R^2}{c_L^2}} = 0,$$

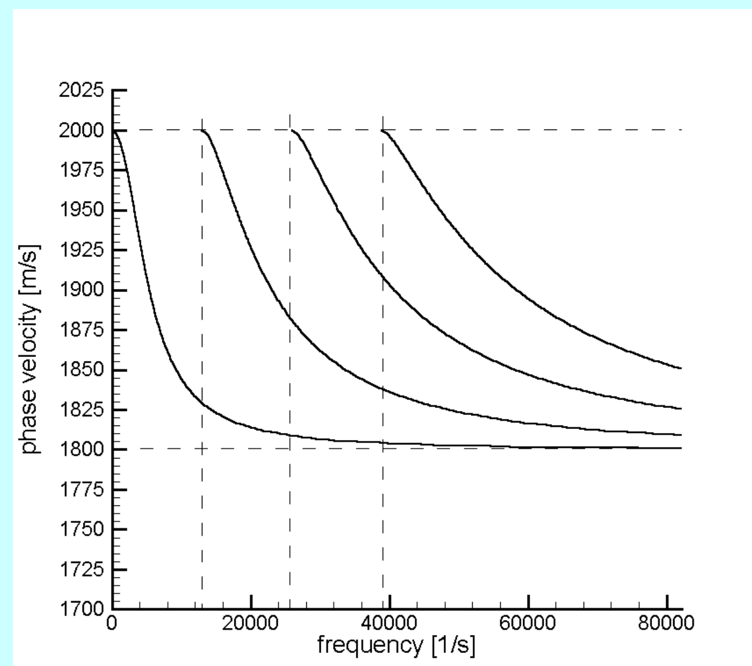
$$c_L^2 := \frac{\lambda + 2\mu}{\rho}, \quad c_T^2 := \frac{\mu}{\rho}.$$

Love wave: phase velocity in a layer of ideal fluid

a) on a rigid foundation with *c=1500 m/s, H=1 m*

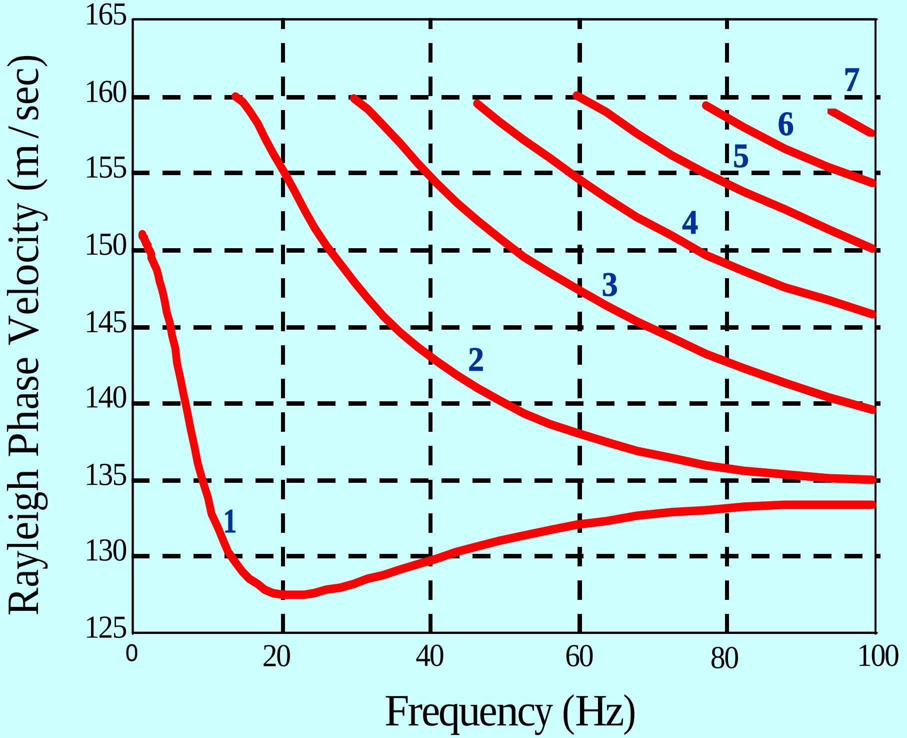
b) on an elastic foundation with *ρ'/ρ=0.8, c=2000 m/s, c'=1800 m/s, H=1 m*





Surface waves in heterogeneous elastic materials

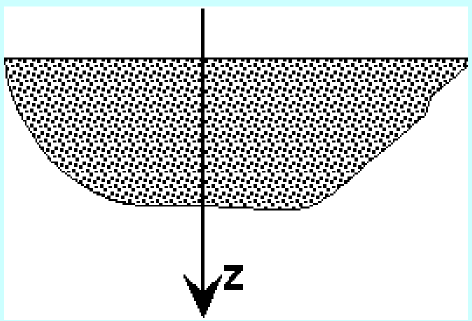
Typical solution of the dispersion relation for Rayleigh waves for a vertically heterogeneous elastic material: existence of infinitely many modes of propagation with critical frequencies



CARLO G. LAI; Simultaneous Inversion of Rayleigh Phase Velocity and Attenuation for Near-surface Site Characterization, PhD-Thesis, Georgia Institute of Technology, 1998.

Main features of surface waves in homogeneous poroelastic media*

Boundary conditions for z=0 :



$$\left(\mathbf{T}^S + \mathbf{T}^S\right) \cdot \mathbf{n} = 0,$$

$$\rho^F \left(\mathbf{v}^F - \mathbf{v}^S\right) \cdot \mathbf{n} = \alpha\left(p^F - n_0 p_{ext}\right),$$

$$\left(\mathbf{v}^F - \mathbf{v}^S\right) - \mathbf{n}\left(\mathbf{v}^F - \mathbf{v}^S\right) \cdot \mathbf{n} = 0.$$

1) $\alpha=0$ – impermeable boundary,

2) $\alpha\neq 0$ – permeable boundary.

A) **Limit of short waves** (high frequency): 1) – two modes, 2) – three modes.
B) **Limit of long waves**: singularity for initial value problem; boundary value problem (far field approximation). 1) – one mode, 2) – two modes.

Inna EDELMAN, K. WILMANSKI, Asymptotic analysis of surface waves at vacuum/porous medium and liquid/porous medium interfaces, *Continuum Mech. Thermodyn.*, **14**, 25-44, 2002
K. WILMANSKI, B. ALBERS; Acoustic waves in porous solid-fluid mixtures, in: K. Hutter, N. Kirchner (eds.), Springer, Berlin (2003, to appear).

Surface waves within the simple model - two-component poroelastic homogeneous

Q=0

Construction of solutions - potentials for displacements:

$$\mathbf{u}^S = grad\,\varphi^S + rot\,\boldsymbol{\psi}^S, \quad \mathbf{v}^S = \frac{\partial \mathbf{u}^S}{\partial t}, \quad \mathbf{e}^S = symgrad\,\mathbf{u}^S,$$

$$\mathbf{u}^F = grad\,\varphi^F + rot\,\boldsymbol{\psi}^F, \quad \mathbf{v}^F = \frac{\partial \mathbf{u}^F}{\partial t}.$$

Ansatz for solution:

$$\varphi^S = A^S(z)\mathrm{E}, \quad \varphi^F = A^F(z)\mathrm{E},$$

$$\psi_z^S = B^S(z)\mathrm{E}, \quad \psi_z^F = B^F(z)\mathrm{E},$$

$$\psi_x^S = \psi_y^S = \psi_x^F = \psi_y^F = 0,$$

$$\rho^S - \rho_0^S = A_\rho^S(z)\mathrm{E},$$

$$\rho^F - \rho_0^F = A_\rho^F(z)\mathrm{E},$$

$$n - n_0 = A^\Delta(z)\mathrm{E}.$$

$$\mathrm{E} := e^{i(kx - \omega t)}.$$

Boundary conditions on the interface vacuum/porous medium:

$$T_{13}\big|_{z=0} = T_{13}^S\big|_{z=0} = \mu^S \left(\frac{\partial u_1^S}{\partial z} + \frac{\partial u_3^S}{\partial x} \right)\bigg|_{z=0} = 0, \qquad \frac{\partial}{\partial t}\left(u_3^F - u_3^S\right)\Big|_{z=0} = 0,$$

$$T_{33}\big|_{z=0} = \left(T_{33}^S - p^F\right)\big|_{z=0} = c_{P1}^2 \rho_0^S \left(\frac{\partial u_1^S}{\partial x} + \frac{\partial u_3^S}{\partial z} \right)\bigg|_{z=0} - 2c_S^2 \rho_0^S \frac{\partial u_1^S}{\partial x}\bigg|_{z=0} - c_{P2}^2\left(\rho^F - \rho_0^F\right)\big|_{z=0} = 0.$$

Dimensionless notation:

$$c_s := \frac{c_S}{c_{P1}}, \quad c_f := \frac{c_{P2}}{c_{P1}}, \quad \pi \to \frac{\pi\tau}{\rho_0^S}, \quad \beta \to \frac{\beta n_0}{\rho_0^S c_{P1}^2},$$

$$r := \frac{\rho_0^F}{\rho_0^S}, \quad z \to \frac{z}{c_{P1}\tau}, \quad k \to k c_{P1}\tau, \quad \omega \to \omega\tau.$$

Boundary value problem in far field approximation,
ω - real, given

Compatibility with field equations and dispersion relation

Independent amplitudes

$$A^F = A_f^1 e^{\gamma_1 z} + A_f^2 e^{\gamma_2 z}, \quad A^F = A_s^1 e^{\gamma_1 z} + A_s^2 e^{\gamma_2 z}, \quad B^S = B_s e^{\zeta z}.$$

with

$$\left(\frac{\zeta}{k}\right)^2 = 1 - \frac{1}{c_s^2}\left(1 + \frac{i\pi}{\omega + i\dfrac{\pi}{r}}\right)\left(\frac{\omega}{k}\right)^2,$$

$$c_f^2\left[\left(\frac{\gamma}{k}\right)^2 - 1\right]^2 + \left[1 + \left(1 + \frac{1}{r}\right)\frac{i\pi}{\omega}\right]\left(\frac{\omega}{k}\right)^4 + \left[1 + c_f^2 + \left(c_f^2 + \frac{1}{r}\right)\right]\left[\left(\frac{\gamma}{k}\right)^2 - 1\right]\left(\frac{\omega}{k}\right)^2 = 0.$$

Eigenvectors

$$\mathbf{R}^1 = \left(B_s, A_s^1, A_f^1\right)^T, \quad \mathbf{R}^2 = \left(B_s, A_s^2, A_f^2\right)^T, \quad A_f^1 = \delta_f A_s^1, \quad A_s^2 = \delta_s A_f^2,$$

$$\delta_f := \frac{1}{r}\frac{\dfrac{i\pi}{\omega}\dfrac{\omega^2}{k^2}}{c_f^2\left[\left(\dfrac{\gamma_1}{k}\right)^2 - 1\right] + \left(\dfrac{\omega}{k}\right)^2 + \dfrac{i\pi}{\omega r}\dfrac{\omega^2}{k^2}}, \quad \delta_s := \frac{\dfrac{i\pi}{\omega}\dfrac{\omega^2}{k^2}}{\left[\left(\dfrac{\gamma_2}{k}\right)^2 - 1\right] + \left(\dfrac{\omega}{k}\right)^2 + \dfrac{i\pi}{\omega r}\dfrac{\omega^2}{k^2}},$$

Boundary conditions

$$\mathbf{AX} = \mathbf{0}, \qquad \mathbf{X} := (B_s, A_f^2, A_s^1)^T,$$

$$\mathbf{A} := \begin{pmatrix} \left(\frac{\zeta}{k}\right)^2 + 1 & 2i\frac{\gamma_2}{k}\delta_s & 2i\frac{\gamma_1}{k} \\ -2ic_s^2\frac{\zeta}{k} & \left[\left(\frac{\gamma_2}{k}\right)^2 - 1 + 2c_s^2\right]\delta_s + rc_f^2\left[\left(\frac{\gamma_2}{k}\right)^2 - 1\right] & \left(\frac{\gamma_1}{k}\right)^2 - 1 + 2c_s^2 + rc_f^2\left[\left(\frac{\gamma_1}{k}\right)^2 - 1\right]\delta_f \\ i\frac{r\omega}{r\omega + i\pi} & -(\delta_s - 1)\frac{\gamma_2}{k} & (\delta_f - 1)\frac{\gamma_1}{k} \end{pmatrix}$$

Exponents in low frequency approximation

singular perturbation

$$\omega << 1:$$

$$\left(\frac{\zeta}{k}\right)^2 = 1 - \frac{r+1}{c_s^2}\left(\frac{\omega}{k}\right)^2, \quad \left(\frac{\gamma_1}{k}\right)^2 = 1 - \frac{r+1}{rc_f^2 + 1}\left(\frac{\omega}{k}\right)^2,$$

$$\left(\frac{\gamma_2}{k}\right)^2 = 1 - \frac{rc_f^4 + 1}{c_f^2(rc_f^2 + 1)}\left(\frac{\omega}{k}\right)^2 - \frac{i\pi}{\omega}\frac{rc_f^2 + 1}{rc_f^2}\left(\frac{\omega}{k}\right)^2,$$

$$\delta_f = 1 - \frac{\omega r}{i\pi}\frac{1 - c_f^2}{1 + rc_f^2}, \quad \delta_f = -rc_f^2\left(1 - \frac{\omega r}{i\pi}\frac{1 - c_f^2}{1 + rc_f^2}\right).$$

High frequency approximation:

$$\mathrm{P}_R\sqrt{1-c_f^2\left(\frac{\omega}{k}\right)^2}+\frac{r}{c_s^4}\left(\frac{\omega}{k}\right)^4\sqrt{1-\left(\frac{\omega}{k}\right)^2}=0,$$

$$\mathrm{P}_R:=\left(2-\frac{1}{c_s^2}\left(\frac{\omega}{k}\right)^2\right)^2-4\sqrt{1-\left(\frac{\omega}{k}\right)^2}\sqrt{1-\frac{1}{c_s^2}\left(\frac{\omega}{k}\right)^2}.$$

For $r=0$ the above relation becomes Rayleigh dispersion relation.

Otherwise two modes:

- leaky Rayleigh wave with the speed smaller than c_s but larger than c_f,
- Stoneley wave with the speed smaller than c_f.

Low frequency approximation:

Dispersion relation (dimensionless!)

$$\left(\frac{\omega}{k}\right)\left[\left(2-\frac{r+1}{c_s^2}\left(\frac{\omega}{k}\right)^2\right)^2-4\sqrt{1-\frac{r+1}{c_s^2}\left(\frac{\omega}{k}\right)^2}\sqrt{1-\frac{r+1}{rc_f^2+1}\left(\frac{\omega}{k}\right)^2}\right]+O\left(\sqrt{\omega}\right)=0.$$

Hence we obtain two solutions:

- Stoneley wave whose speed of propagation is of the order $O\left(\sqrt{\omega}\right)$
- Rayleigh wave whose speed of propagation follows from the equation:

$$\left(2-\frac{c_{P1}^2}{c_{oS}^2}\left(\frac{\omega}{k}\right)^2\right)^2-4\sqrt{1-\frac{c_{P1}^2}{c_{oS}^2}\left(\frac{\omega}{k}\right)^2}\sqrt{1-\frac{c_{P1}^2}{c_{oP1}^2}\left(\frac{\omega}{k}\right)^2}=0.$$

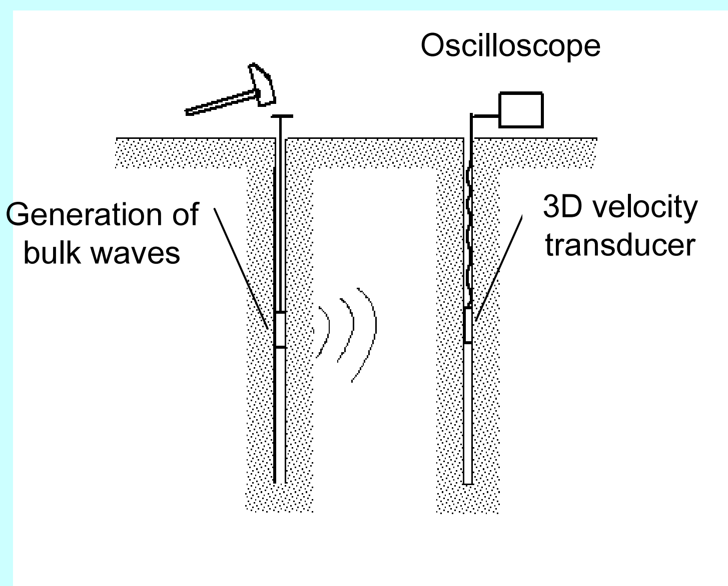
6) Applications of waves in testing of soils

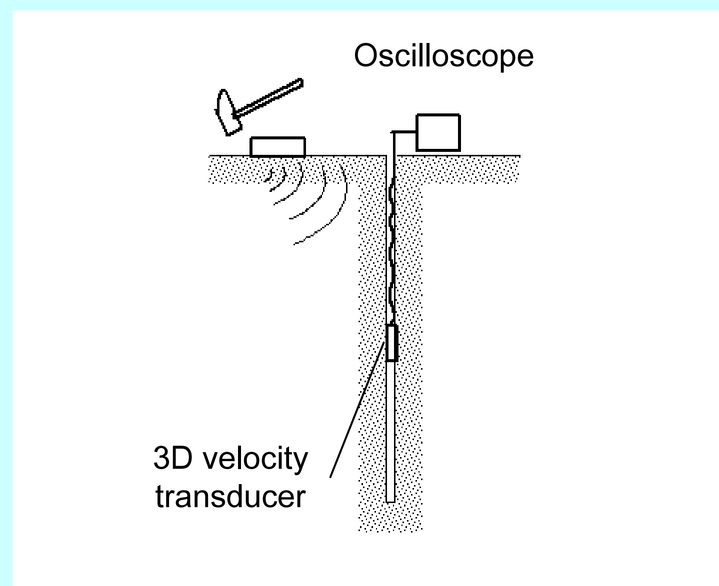
General principles:

destructive testing, the original soil structure must be damaged, e.g.

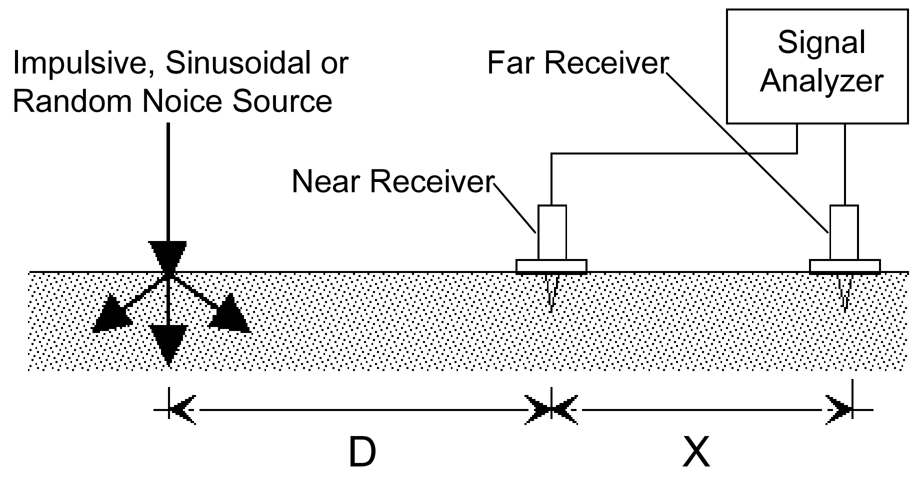
- laboratory testing
- acoustic testing from boreholes, for example - explosive

Cross-hole and down-hole acoustic tests

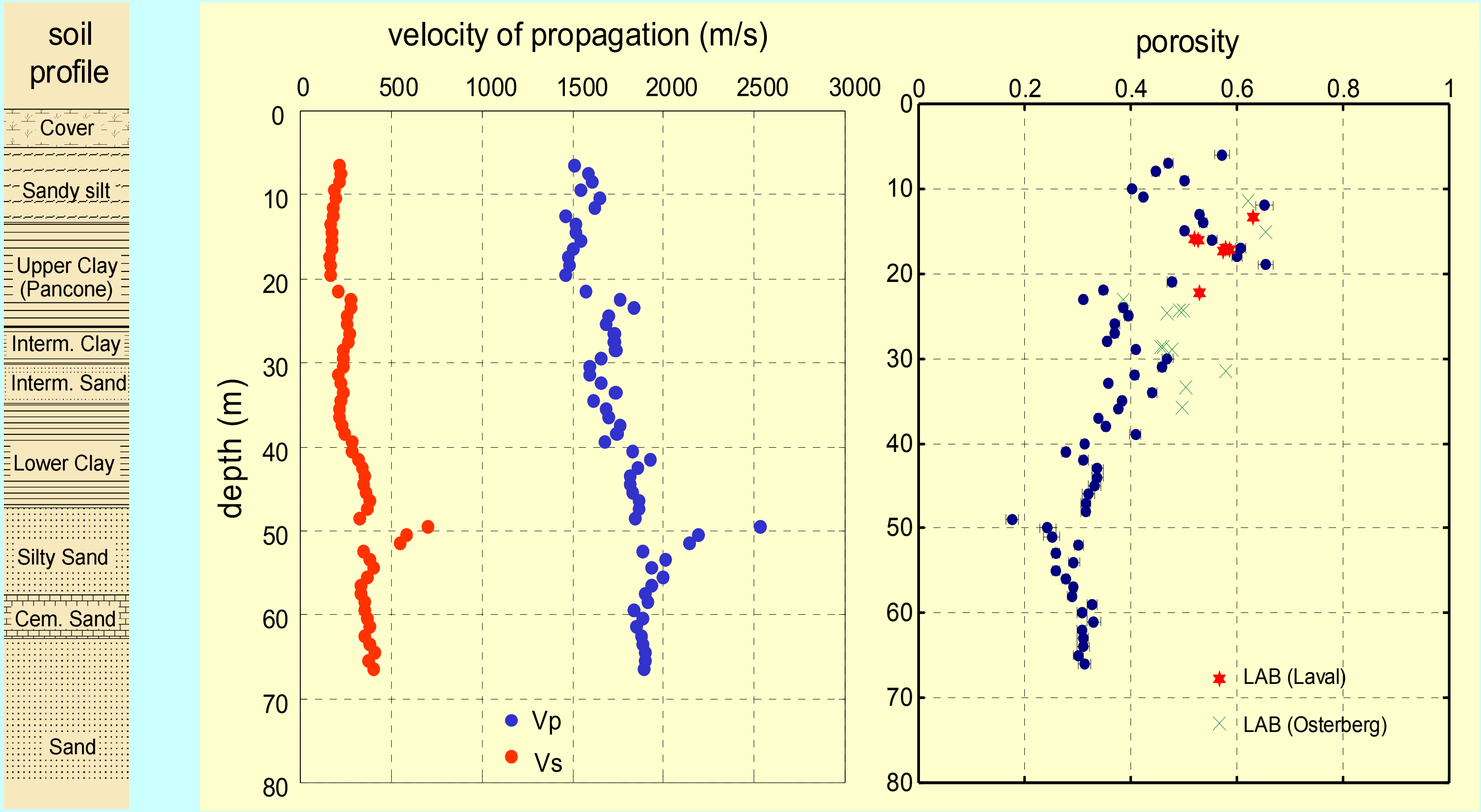




nondestructive testing: surface waves, the original structure of the soil is not influenced, e.g. SASW (Spectral Analysis of Surface Waves); different configurations possible (e.g. multistations)



Experimental measurements: c_{P1}, c_S and porosity profiles



Pisa site. After: FOTI,S., LAI, C., LANCELLOTTA, R.; Porosity of Fluid-saturated Porous Media from Measured Seismic Wave Velocities, Géotechnique, Vol. 52, No. 5, 359-373 (2002).

LAI, C.; Recent Advances in the Solution of Some Parameter-Identification Problems Relevant To Soil Dynamics, Lecture, Oct. 21, 2002, ROSE School, Pavia, Italy.

Pieter Brueghel the Elder (1560), *Temperantia*

- question of education, reading, writing, calculation, geometry, singing lecturing, astronomy, geography.