**Mechanics of Solids and Structures.**
**In memory of Professor Margaret Woźniak**

# A FEW REMARKS ON MICRO/MACRO TRANSITIONS AND GASSMANN RELATIONS FOR POROELASTIC MATERIALS

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## INTRODUCTION

Linear poroelastic models of granular and porous materials are constructed primarily for the description of acoustic and seismic waves in soils. Such models are rather useless in foundation engineering design where irreversible deformations must be incorporated. Acoustic waves used in testing of soils possess a small amplitude and, for this reason, their modeling with linear equations yields very good results (e.g. compare [3], [4]). However, in order to evaluate experimental results one has to know relations between amplitudes, speeds of propagation and attenuation of waves and morphological properties of the material such as porosity, degree of saturation and tortuosity. In addition, it is useful to connect macroscopic parameters of the model with real (true) properties of components. This concerns, in particular, partial mass densities, macroscopic compressibility moduli, coupling parameters on the one hand-side, and real (true) mass densities, real compressibility parameters and, again, porosity, on the other hand. Such relations for a very simple static model of soils were derived by Gassmann ([1]).For the famous Biot model, similar considerations were presented by Biot and Willis ([2]). These considerations are purely macroscopic and heuristic. It was only very recently that a systematic micro/macro derivation was constructed ([7]). Similar ideas as these in the last quoted paper can be also found in the work of Abousleiman and Cheng ([8]).

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It should be stressed that a systematic derivation of micro/macro relations for parameters of the linear model could be as yet constructed only for bulk compressibilities. The shear modulus, particularly for sands and similar granular materials, must be constructed in a different way. A statistical approach, even though rather sophisticated, was not very successful (e.g. [3]) and the problem remains unsolved. In many practical applications, one tries to overcome the problem by changing the set of macroscopic parameters in such a way that the Poisson number rather than the shear modulus is being used and the former can be considered to be independent of porosity for many soils (see: [5]). Recently, an attempt is made to extend the procedure presented in ([7]) of constructing micro/macro relations on problems of wave propagation in unsaturated materials (see: [11]). This extension is complex and very cumbersome and requires a clear understanding of each step in the procedure. The purpose of this work is the presentation of the "flow chart" for the two-component model as a guiding line for more complex models.

We use the two-component linear model of poroelastic material following from general thermodynamical considerations (e.g. [6]). This model reminds Biot's model but it contains an additional material parameter describing an influence of spatial changes of porosity on partial stresses.

## 2. GOVERNING EQUATIONS OF THE MODEL

The model is based on the assumption that a porous material is described by a two-component mixture-like continuum. We consider isothermal processes and this limitation means that the following set of fields is chosen for the description of a porous medium:

1) partial mass densities of the solid skeleton and of the fluid respectively, $\rho^S, \rho^F$,

2) partial velocities of the solid skeleton and of the fluid respectively, $\mathbf{v}^S, \mathbf{v}^F$,

3) porosity, $n$.

All these fields are defined on a common domain $\mathcal{B}$ of the porous medium, and are functions of time. Sufficient smoothness assumptions are made which we do not discuss here in details.

Field equations for these fields follow from the balance equations whose linear form is as follows

1) partial mass balance equations

$$\frac{\partial \rho^S}{\partial t} + \rho_0^S \operatorname{div} \mathbf{v}^S = 0, \quad \frac{\partial \rho^F}{\partial t} + \rho_0^F \operatorname{div} \mathbf{v}^F = 0, \tag{1}$$

2) partial momentum balance equations

$$\rho_0^S \frac{\partial \mathbf{v}^S}{\partial t} = \text{div } \mathbf{T}^S + \hat{\mathbf{p}}, \quad \rho_0^F \frac{\partial \mathbf{v}^F}{\partial t} = \text{div } \mathbf{T}^F - \hat{\mathbf{p}}, \tag{2}$$

3) porosity balance equation

$$\frac{\partial \Delta_{\mathbf{n}}}{\partial t} + \Phi \text{ div}\left(\mathbf{v}^F - \mathbf{v}^S\right) = \hat{n}, \quad \Delta_n = n - n_E. \tag{3}$$

In these equations, $\rho_0^S, \rho_0^F$ are initial (constant) partial mass densities, $\mathbf{T}^S, \mathbf{T}^F$ are partial (Cauchy) stress tensors in the solid skeleton and the fluid, respectively, $\hat{\mathbf{p}}$ is momentum source while $\hat{n}$ is porosity source, $\Phi$ is a material parameter of the porosity flux, and $n_E$ is the porosity in thermodynamical equilibrium. As we neglect temperature changes the latter appears when there is no diffusion, i.e. for $\mathbf{v}^S = \mathbf{v}^F$.

We assume the medium to be linearly isotropic which means that macroscopic constitutive relations have the following form

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S e\mathbf{1} + 2\mu^S \mathbf{e}^S + Q\varepsilon\mathbf{1} + \beta\Delta_n \mathbf{1} - N\left(n - n_0\right)\mathbf{1},$$

$$\mathbf{T}^F = -p^F \mathbf{1}, \quad p^F = p_0^F - p_0^F \kappa\varepsilon - Qe + \beta\Delta_n - N\left(n - n_0\right),$$

$$n_E = n_0\left(1 + \delta e\right), \tag{4}$$

$$\hat{\mathbf{p}} = \pi\left(\mathbf{v}^F - \mathbf{v}^S\right),$$

$$\hat{n} = -\frac{\Delta_n}{\tau}.$$

In these relations, $\mathbf{T}_0^S, p_0^F, n_0$ are the initial partial stress in the skeleton, the initial partial pressure in the fluid, and the initial porosity, respectively. The quantities

$$e = tr \mathbf{e}^S = \frac{\rho_0^S - \rho^S}{\rho_0^S}, \quad \varepsilon = \frac{\rho_0^F - \rho^F}{\rho_0^F}, \tag{5}$$

describe partial volume changes of the skeleton and of the fluid, respectively. Hence, the model contains the following set of macroscopic material parameters

$$\lambda^S, \ \mu^S, \kappa, Q, N, \delta, \beta, \pi, \tau, \Phi. \tag{6}$$

Obviously, the first two parameters are Lamé constants of the two-component model, $\kappa$ is the macroscopic compressibility of the fluid, $Q, N$ are coupling constants, $\delta$ describes equilibrium changes of porosity, and $\Phi, \beta, \pi, \tau$ are nonequilibrium parameters related to the porosity flux, the coupling of stresses to nonequilibrium porosity changes, to diffusion (proportional to the classical diffusion coefficient) and to relaxation properties of porosity. All these parameters are functions of the initial porosity $n_0$ and some other initial quantities. We return later to this point.

The micro/macro transition procedure which we discuss in this paper is based on the analysis of an initial state and of a current state which may vary only due to stationary processes (i.e. constant fluxes). It does not refer to any temporal behavior of the system. Such an approach is possible if the system does not possess a memory. It can be shown (e.g. [9]) that the only source of memory in this model is the relaxation of porosity which means that, for the purpose of this paper, we assume

$$\tau \to \infty \tag{7}$$

Then the porosity balance equation can be easily integrated. Substitution of mass balance equations (1) in (3) yields

$$\frac{\partial}{\partial t}\left(\Delta_n - \Phi\left(\frac{\rho^F}{\rho_0^F} - \frac{\rho^S}{\rho_0^S}\right)\right) = 0 \quad \Rightarrow \quad \Delta_n = \Phi(e - \varepsilon), \tag{8}$$

or, equivalently,

$$n = n_0\left(1 + \delta e + \gamma(e - \varepsilon)\right), \quad \gamma = \frac{\Phi}{n_0}. \tag{9}$$

Simultaneously, the partial mass balance equations imply that in the thermodynamical equilibrium

$$\mathbf{v}^S\big|_E = \mathbf{v}^F\big|_E \quad \Rightarrow \quad e = \varepsilon. \tag{10}$$

Bearing the above remarks in mind, we can skip the porosity from the list of fields.

In order to simplify our considerations, we neglect also the coupling parameter $\beta$

$$\beta = 0. \tag{11}$$

It can be shown that this is indeed justified in linear models. Parameter $\beta$ plays an important role in the analysis of nonlinear waves but it does not influence sound propagation in porous media (see: [10]).

There exists a good deal of confusion concerning the denotation of material parameters. In this paper we use also the following combinations

$$K = \lambda^S + \frac{2}{3}\mu^S, \quad v, \quad C = \frac{1}{n_0}\left(Q + \rho_0^F \kappa\right), \quad M = \frac{\rho_0^F \kappa}{n_0^2}, \quad N, \quad \pi, \quad \delta, \quad \gamma. \tag{12}$$

Obviously, $K$ is the bulk compressibility modulus and $v$ is the Poisson number.

Further, we consider only spherically symmetric configurations in which parameters $v$ and $\pi$ do not appear. The list of independent fields reduces to $e, \varepsilon$ and the conjugate partial pressures satisfy the following *macroscopic constitutive relations*

$$p^S - p_0^S = -Ke - Q\varepsilon + N(n - n_0), \quad n - n_0 = n_0[\delta e + \gamma(e - \varepsilon)], \tag{13}$$

$$p^F - p_0^F = -p_0^F \kappa\varepsilon - Qe - N(n - n_0), \quad p^S = -\mathrm{tr}\mathbf{T}^S.$$

## 3. MICRO/MACRO TRANSITION PROCEDURE

In theories of porous materials macroscopic models are related to microscopic relations of such media by means of the volume averaging. By the choice of a domain of averaging for each point x of the macroscopic domain of the porous medium and at the instant of time $t$ which is called the representative elementary volume, $REV(\mathbf{x}, t)$, one constructs macroscopic fields by the integration over this domain. Obviously, $REV$ must be sufficiently small from the macroscopic point of view and sufficiently large from the microscopic point of view. The former means that all properties described locally by microscopic fields are prescribed to a single macroscopic point x and this usually yields the necessity of introducing additional macroscopic fields such as porosity, degree of saturation or tortuosity.

The latter means, for example, that microscopic properties may be consider as random. On the microscopic level of the two-component system (a saturated porous medium), the domain $REV$ consists of two nonoverlapping subdomains

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characterized by the function $\chi(\mathbf{z},t)$ which is zero for the point $\mathbf{z} \in REV(\mathbf{x},t)$ if this point is occupied by the true skeleton and it is one if this point is occupied by the true fluid. By means of this function one extends real (true) fields of mass densities, momenta, etc. on the whole domain. These extensions are continuous but otherwise arbitrary.

In the case of mass densities the construction of volume averages is as follows

$$\rho^S(\mathbf{x},t) = \frac{1}{\text{vol}(REV(\mathbf{x},t))} \int_{REV(\mathbf{x},t)} \rho^{SR}(\mathbf{z},t)\big(1-\chi(\mathbf{z},t)\big)\, dV_z \;,$$

$$(14)$$

$$\rho^F(\mathbf{x},t) = \frac{1}{\text{vol}(REV(\mathbf{x},t))} \int_{REV(\mathbf{x},t)} \rho^{FR}(\mathbf{z},t)\,\chi(\mathbf{z},t)\, dV_z \;,$$

where $p^{SR}, p^{FR}$ are extensions of real (true) mass densities of the skeleton and of the fluid. At points which are indeed occupied by the corresponding material these mass densities are such as they were in the one-component situations. Extensions on parts of the domain in which this material is not present are finite but otherwise arbitrary. They do not contribute to macroscopic quantities due to the definition of the characteristic function $\chi$. If variations of real mass densities within $REV$ are small then one can write approximately (the zeroth order approximation)

$$\rho^S(\mathbf{x},t) = \big(1-n(\mathbf{x},t)\big)\rho^{SR}(\mathbf{x},t), \quad \rho^{SR}(\mathbf{x},t) = \rho^{SR}(\mathbf{z}_S,t),\; \mathbf{z}_S \in REV(\mathbf{x},t),$$

$$\rho^F(\mathbf{x},t) = n(\mathbf{x},t)\rho^{FR}(\mathbf{x},t),\; \rho^{FR}(\mathbf{x},t) = \rho^{FR}(\mathbf{z}_F,t),\; \mathbf{z}_F \in REV(\mathbf{x},t), \quad (15)$$

$$n(\mathbf{x},t) = \frac{1}{\text{vol}(REV(\mathbf{x},t))} \int_{REV(\mathbf{x},t)} \chi(\mathbf{z},t)\, dV_z \;,$$

where $\mathbf{z}_S$ is an arbitrary point of the skeleton within $REV(\mathbf{x},t)$ and $\mathbf{z}_F$ an arbitrary point of the fluid within $REV(\mathbf{x},t)$. Obviously, the last relation defines the porosity.

Similar relations hold for the initial values of mass densities

$$\rho_0^S = (1-n_0)\rho_0^{SR}, \quad \rho_0^F = n_0\,\rho_0^{FR} \;. \tag{16}$$

If we introduce real volume changes rather than real mass densities then we have for the skeleton and for the fluid, respectively,

$$e^R = \frac{\rho_0^{SR} - \rho^{SR}}{\rho_0^{SR}}, \quad \varepsilon^R = \frac{\rho_0^{FR} - \rho^{FR}}{\rho_0^{FR}} \;. \tag{17}$$

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Simple combination of the above relations and the linearity of the model, i.e.

$$\max\left\{|e|, |\varepsilon|, \left|e^R\right|, \left|\varepsilon^R\right|\right\} << 1 \tag{18}$$

yields the following *kinematic compatibility conditions*

$$e = e^R + \frac{n - n_0}{1 - n_0}, \quad \varepsilon = \varepsilon^R - \frac{n - n_0}{n_0}. \tag{19}$$

Corresponding definitions of momenta are less obvious due to their vectorial character. We skip here detailed considerations and present further only those results which are essential for our problem of determination of some material parameters. Even more complicated are constructions of averages of fluxes. We assume that the transition for the momentum fluxes reduced to the contributions of pressures has the form

$$p^S = (1 - n)p^{SR}, \quad p_0^S = (1 - n_0)p_0^{SR},$$

$$p^F = np^{FR}, \quad p_0^F = n_0\ p_0^{FR}, \tag{20}$$

where $p^{SR}$ is the real current pressure in the skeleton, $p^{FR}$ is the real current pressure in the fluid (called also the *pore pressure*), etc. These relations are far less obvious than (15), (16) and do not account for cohesive forces, capillary pressure, surface tension, etc. At the best, we may consider them as a rough approximation. We accept this approximation for the purpose of this work.

Combination of relations (20) yields in the linear model the following *dynamical compatibility conditions*

$$p^S - p_0^S = (1 - n_0)\left(p^{SR} - p_0^{SR}\right) - (n - n_0)p_0^{SR},$$

$$p^F - p_0^F = n_0\left(p^{FR} - p_0^{FR}\right) + (n - n_0)p_0^{FR}. \tag{21}$$

In addition to the above relations, we assume that both real components have definite mechanical properties. For pressures they have the form of the following *microscopic constitutive relations*

$$p^{SR} - p_0^{SR} = -K_s e^R, \quad p^{FR} - p_0^{FR} = -K_f \varepsilon^R, \tag{22}$$

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where $K_s$ is the real compressibility of the skeleton and $K_f$ of the fluid. Let us note that for gases the second relation must hold for pressures and not only for pressure differences. This property yields certain difficulties for unsaturated materials (see: [11]) which is immaterial for our considerations.

## 4. FUNDAMENTAL EQUATIONS FOR COMPRESSIBILITIES

Let us collect the unknown quantities of a spherical problem which we have introduced above for both levels of description. They form the following set of 9 functions

$$\left\{ e, \varepsilon, e^R, \varepsilon^R, p^S - p_0^S, p^F - p_0^F, p^{SR} - p_0^{SR}, p^{FR} - p_0^{FR}, n - n_0 \right\}. \quad (23)$$

Simultaneously, we have 9 homogeneous equations at the disposal: three macroscopic constitutive relations (13), two kinematic compatibility relations (19), two dynamical compatibility relations (21) and two microscopic constitutive relations (22). As these equations are homogeneous with respect to the fields (23), we must impose a condition of the zero value of the determinant of this set. We proceed in a different way. In order to see clearly the situation, we repeat all equations

$$e = e^R + \frac{n - n_0}{1 - n_0}, \quad \varepsilon = \varepsilon^R - \frac{n - n_0}{n_0},$$

$$p^S - p_0^S = (1 - n_0)\left( p^{SR} - p_0^{SR} \right) - (n - n_0) p_0^{SR},$$

$$p^F - p_0^F = n_0 \left( p^{FR} - p_0^{FR} \right) + (n - n_0) p_0^{FR}, \quad (24)$$

$$p^S - p_0^S = - K e - Q \varepsilon + N(n - n_0), \quad p^F - p_0^F = - p_0^F \kappa \varepsilon - Q e - N(n - n_0),$$

$$p^{SR} - p_0^{SR} = - K_s e^R, \quad p^{FR} - p_0^{FR} = - K_f \varepsilon^R, \quad n - n_0 = n_0 [\delta e + \gamma(e - \varepsilon)].$$

By the linear combination of the first 8 equations we obtain the following relation for the changes of porosity

$$n - n_0 = - \frac{1}{(K_s - K_f) - \left( p_0^{SR} - p_0^{FR} \right)} \left\{ [(K + Q) - (1 - n_0) K_e] e + \right.$$
$$\left. + \left[ (p_0^F \kappa + Q) - n_0 K_f \right] \varepsilon \right\}. \quad (26)$$

Instead of the condition on the determinant of the set (24) we can require that the last equation (24) is a linear combination of the remaining 8 equations. Consequently, we impose the condition that $(24)_9$ and (26) are identical. If fully exploited this yields a single condition for material parameters. We strengthen this condition and require that coefficients of $e$ and $. \, \varepsilon$ are identical in both relations. This sufficient condition for the existence of solutions of the set (24) yields the following relations

$$\delta = \frac{K_V - K}{n_0 \left( K_s - K_f \right)}, \quad \Phi = n_0 \, \gamma = \frac{\left( p_0^F \, \kappa + Q \right) - n_0 \, K_f}{K_s - K_f} ,$$

$$K_V = (1 - n_0) K_S + n_0 K_f .$$

$$(27)$$

We have neglected the influence of the difference $p_0^{SR} - p_0^{FR}$ as this is, obviously, much smaller than $K_s - K_f$.

The remaining equations can be solved with respect to a chosen unknown. We choose

$$\Delta p = \left( p^S - p_0^S \right) + \left( p^F - p_0^{FR} \right). \tag{28}$$

Then it follows for macroscopic volume changes

$$e = \frac{a_{12} - a_{22}}{a_{11} a_{22} - a_{12} a_{21}} \Delta p , \quad \varepsilon = \frac{a_{21} - a_{11}}{a_{11} a_{22} - a_{12} a_{21}} \Delta p , \tag{29}$$

where

$$a_{11} = (1 - n_0) K_s + \left( K_s - K_f \right) \frac{K - Q - (1 - n_0) K_s}{K_s + K_f - 2N} ,$$

$$a_{12} = n_0 \, K_f - \left( K_s - K_f \right) \frac{p_0^F \kappa - Q - n_0 K_f}{K_s + K_f - 2N} , \tag{30}$$

$$a_{21} = K + Q ,$$

$$a_{22} = p_0^F \, \kappa + Q .$$

Substitution in the last relation (24) yields the formula for $n - n_0$ while the subsequent substitution in the remaining relations (24) yields formuli for $e^R, \varepsilon^R, p^{SR} - p_0^{SR}, p^{FR} - p_0^{FR}$. All of them contain the linear homogeneous dependence on $\Delta p$.

Field equations are in the present case satisfied identically because the above fields are time and space independent and velocity fields are identically zero. In order to find the solution of the problem we have to formulate the boundary conditions. It is easy to check that due to the symmetry the system requires two such conditions. One of them is common for all cases which we consider further in this work. Namely, we assume that the system is loaded by the increment of the external pressure. As the system must be in equilibrium with the pressure carried by both component we have the *following equilibrium condition*

$$\left(p^S - p_0^S\right) + \left(p^F - p_0^F\right) = \Delta p, \tag{31}$$

where $\Delta p$ is now a given change of external pressure.

Consequently, we obtain the explicit solution of the problem – volume changes $e$ and $\varepsilon$ as well as all other quantities are given in terms of the known increment $\Delta p$. However, this solution is only formal. It is easy to see that relations (29), (30) contain both the macroscopic parameters $K, p_0^F \kappa, Q, N$ as well as the microscopic compressibilities $K_s, K_f$ and the porosity $n_0$. Hence, we must find conditions which reduce this set of parameters. This is done by the second boundary condition, formulated for various boundaries. We discuss this problem in the next Section.

## 5. GEDANKENEXPERIMENTS

Gedankenexperiments are theoretical considerations referring to experiments which are in principle possible but usually require ideal conditions difficult to achieve in laboratories. Biot and Willis [2] proposed three such experiments which would yield a solution of problem presented at the end of the previous Section:

1) *Jacketed undrained experiment*, performed on a sample of a two-component porous material, for which locally in time the following condition must be satisfied

$$\oint_{\delta V} \left( \mathbf{v}^F - \mathbf{v}^S \right) \cdot \mathbf{n} \, dS == \int_V \text{div} \left( \mathbf{v}^F - \mathbf{v}^S \right) dV = 0 \implies$$

$$\implies e = \varepsilon \tag{32}$$

where $V$ is the volume of the sample and $\delta V$ its surface. This means that the fluid remains within the sample and, consequently, the homogeneity yields the condition of equal macroscopic volume changes.

2) *Jacketed drained experiment*, for which the fluid may flow through the boundary of the sample to the external world. This yields the condition that the external loading cannot change the pore pressure

$$p^{FR} = p_0^{FR} \tag{33}$$

3) *Unjacketed experiment* in which a heap of the porous material is immersed in the fluid which, in turn, carries to the sample the external change of pressure $\Delta p$. Consequently, the pore pressure must change on the same amount

$$p^{FR} - p_0^{FR} = \Delta p . \tag{34}$$

Substitution of the results of the previous Section in these conditions yields three relations between material parameters. For instance, in the first case, we have

$$a_{12} - a_{22} = a_{21} - a_{22} . \tag{35}$$

In addition to these relations, one usually assumes that in the second experiment, jacketed drained, one can indeed measure the so-called *drained compressibility modulus* $K_d$, defined by the relation

$$K_d = -\frac{\Delta p}{e} . \tag{36}$$

This gives rise to the fourth relation. Hence, we obtain four equations for four unknown macroscopic material parameters: $K, p_0^F \kappa, Q, N$. They are given in terms of $K_s, K_f$ and $n_0$. We have shown in [7] that these nonlinear equations possess two solutions. One of them yields the coupling parameter $N$ unequal zero and the order of magnitude of this parameter is the same as $Q$. There exists as well

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the solution in which $N = 0$. This solution is identical with the classical Gassmann relations

$$K = \frac{(K_s - K_d)^2}{\dfrac{K_s^2}{K_w} - K_d}, \quad C = \frac{1}{n_0}\left(Q + p_0^F \kappa\right) = \frac{K_s(K_s - K_d)}{\dfrac{K_s^2}{K_w} - K_d}$$

$$M = \frac{\rho_0^F \kappa}{n_0^2} = \frac{K_s^2}{\dfrac{K_s^2}{K_w} - K_d}, \quad K_w = \left[\frac{1 - n_0}{K_s} + \frac{n_0}{K_f}\right]^{-1}.$$

(37)

We skip here the presentation of numerical examples which can be found in the paper [7]. However, it should be stressed that in the range of parameters appearing for soils saturated with water the compressibilities $K$ and $M$ are of more than one order of magnitude bigger than coupling parameters $Q, N$ and that sum $Q + N$ for the solution with $N \neq 0$ is approximately equal to $Q$ for the solution with $N = 0$. These are important conclusions for the linear wave analysis of poroelastic materials.

## 6. CONCLUDING REMARKS

The above presented "flow chart" of the micro/macro transition for saturated porous materials is rather straightforward. It consists of the solution of a homogeneous "field" problem and the subsequent use of boundary conditions following from Gedankenexperiments. In cases of multicomponent systems it becomes, for many reasons, much more complicated. The main problem is the formulation of boundary conditions. These are complex because the flow of two different components through the boundary requires some additional assumptions on the physical structure of the boundary. Simultaneously, as the number of unknowns is much higher – in addition to these which appear for two components we have the compressibility of the second fluid component and various coupling parameters – the analytical solution seems to be almost impossible and the numerical analysis requires a particular aptitude. The work [11] contains the first results in this direction.

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